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Graviton and scalar two-point functions in a CDL background for general dimensions

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Abstract: We compute the two-point functions of the scalar and graviton in a Coleman-De Luccia type instanton background in general dimensions. These are analytically continued to Lorentzian signature. We write the correlator in a form convenient for examining the "holographic" properties of this background inspired by the work of Freivogel, Sekino, Susskind and Yeh(FSSY). Based on this, we speculate on what kind of boundary theory we would have on this background if we assume that there exists a holographic duality.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Models of Quantum Gravity, Solitons Monopoles and Instantons

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1 Introduction

The AdS/CFT correspondence [1] has shed new light on how to think about gauge theories and gravity theories in general. It is natural to try to find such a way of understanding gravity with a background we live in, namely, in deSitter space. There have been many ideas put forth on how to think about such matters [2–8]. The current view expressed in the literature is that since a boundary theory of such a correspondence would have to lie at timelike infinity, one would have to take into account bubble nucleation for realistic theories. The argument is that if we have finite probability for bubble nucleation(which is very possible by semi-classical arguments such as [9]) for any path we take to future timelike infinity, we must encounter some kind of bubble nucleation along the way.

Thus it is natural to consider quantum gravity in backgrounds with bubble nucleation, for example that described by a Coleman-De Luccia(CDL) instanton [10]. The Penrose diagram of this instanton (the 'bounce' as they put it) is shown in figure 1. If we consider the timelike flat region(region A) of this background in D dimensions, it has a well defined spatial infinity at Σ, which is a S^{D-2}.

Freivogel, Sekino, Susskind, and Yeh proposed that there may well be some kind of holographic correspondence between the bulk theory in region A and its boundary Σ in [5]. A further interpretation of the consequences of this calculation was pursued in [6]. In these papers, they have proposed that in 4 dimensions, the holographic dual living at Σ corresponding to the bulk gravity theory would be a Liouville theory. Furthermore, they have identified the conformal time coordinate with the Liouville field on the boundary. If this were true, time in the bulk would be emergent from a Liouville field on the boundary.

This was suggested by writing out the two point functions in a manner that made the (potentially) holographic structure more evident and analyzing relevant pieces that showed up in this propagator. What we will do in this paper is to further carry out this analysis to higher dimensions.

We will do this by obtaining the two point function of the transverse traceless graviton and scalar in this background. This has been done in the past in 4 dimensions [5, 11], but we extend the calculation to general dimensions.

This calculation is meaningful in three ways. First, doing a 'holographic expansion' of the propagators in a general dimensional background gives a clear framework as to how to do such an expansion in the 4 dimensional case. Having an explicit D in the expansion helps organizing the terms in the expression for the propagators.

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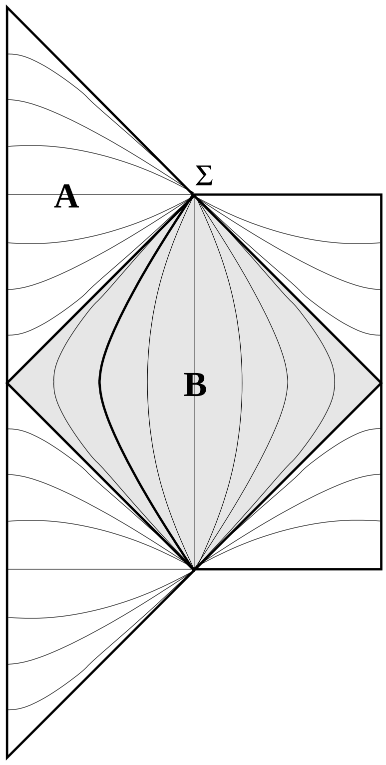

Figure 1. The Penrose diagram for the Coleman-De Luccia instanton. The bold curve in the grey region is the bubble wall. The space is flat to the left of the wall, and deSitter to the right side of the wall.

Also, exploring a potential holographic duality of this kind in general dimensions turns out to be an interesting topic in itself. It would be very interesting to see which statements FSSY have set forth for the 4 dimensional case still hold in general dimensions. As we shall see, all of their conjectures regarding the existence of a holographic duality could be repeated with slight modification for the general dimensional case. In fact, the clearer organization of the terms in the holographic expansion enables us to say a bit more.

Last but not least, we expect gravity in odd and even dimensions to behave very differently, and it will be interesting to see how this shows in the propagator. We will be able to see that if such a correspondence existed, the boundary theory would in fact be very different for odd and even dimensions.

In this paper, we will consider a theory with gravity and a single scalar field with a potential that has two minima in a general dimensional space. We assume the thin wall limit can be applied, that is, that there is a classical solution of the theory where we have two distinct regions of space with different cosmological constants seperated by a thin wall. We will be interested in the case where we have a flat space inside the bubble and a de Sitter space outside. We will review this background — the CDL instanton — in section 2.

The thin wall introduces a boundary in our space. We will calculate two point functions of the transeverse traceless graviton field and the scalar field inside the timelike region of the thin wall, and take it to the infinite boundary of the thin wall(which is a S^{D-2}) and see how it behaves.

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We will do this for the graviton two point function in section 3. We will follow the standard procedure of first calculating the two point function of the graviton on the Euclidean instanton, and then analytically continuing it to Lorentzian signature [12]. After that, we will write this as a sum of massive tensor field propagators in H^{D-1}. We will not be interested in the ordinary part of the propagator, but the part of the propagator that arises due to the existence of the wall.

As pointed out in [11, 13] the propagator obtained would still have some residual gauge freedom we have to project out. We will explain in section 4.1 the 'naive way' of projecting out those degrees of freedom. We will obtain the propagator after this projection in section 4.2. Finally, in section 4.3 we will point out the subtlety overlooked by the method of projection employed in section 4.1 and present the final gauge-fixed two point function.

We will summarize the graviton two point function and examine important features we see in it in section 5.

A similar calculation for the scalar is done in section 6. The final 'holographic expansion' for the scalar is written out in section 6.7 when the scalar is massless and in section 6.8 for the general case. A major difference of the scalar case with the graviton case is pointed out in the latter section as well.

Finally, we will interpret the results in section 7. In this section, we try to guess what the theory living on the boundary S^{D-2} would look like if we assume a holographic correspondence, as there are a number of interesting proposals we could make just from the 'holographic expansion' of the propagators.

We first propose the conformal structure of the theory living on the boundary, and argue that it is highly possible that it contains gravity. We also propose a possible holographic correspondence between fields in the bulk with operators on the boundary. We see how the tunable dimensionful parameter of the theory, namely the wall position, plays a role in this correspondence. We pay special attention to the operators whose conformal dimensions depend on the wall position. We mention that the number of these are finite in even dimensions, while they are infinite in odd dimensions. Based on the behavior of these operators, we note that in odd dimensions, some kind of phase transition seems to happen as the dimensions of infinitely many operators become complex at a critical position of the wall.

As one of the objectives of the paper is to present a thorough description of the calculation procedure of the graviton two point function, there are many technical details that might be uninteresting to some readers. I believe reading section 2 for understanding the instanton background we are working in, and sections 5 and 7 for seeing the results and implications of the calculation would be enough for those who wish to skip such details.

2 The background

We consider a theory with gravity and a single scalar field ϕ with a potential $V(\phi)$, in a D dimensional space. We assume D is even.

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By setting $\kappa = 8\pi G = 1/2$, our Lagrangian would look like,

$$S = \frac{1}{2}\int d^D x\sqrt{-g}(-g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2V(\phi) + 2R) \tag{2.1}$$

where we use the sign convention, $(- + + \cdots +)$. We wish to obtain a classical solution for this action, and expand around that background. Now V in general could be anything, but we are interested in the situation of tunneling between ground states with positive and zero scalar vacuum expectation values. Hence we assume that V has two local minima, each at ϕ_+, ϕ_-, with $V(\phi_+) > V(\phi_-) = 0$.

We may follow the course of Euclideanizing the action, solving for it, then analytically continuing it. Also, we assume an $O(D-1)$ symmetry of the solution for Euclidean metric, and furthermore assume that the classical solution for ϕ is only a function of the radial coordinate. This symmetry may not exist for all classical solutions, but we are not interested in cases that do not respect this symmetry. So we may begin by setting the metric of the D dimensional Euclidean manifold as,

$$ds^2 = dt^2 + a(t)^2(d\theta^2 + \sin^2\theta d\Omega_{D-2}^2) \tag{2.2}$$

Then we obtain the classical solution by solving,

$$\ddot{\phi} + (D-1)\frac{\dot{a}}{a}\dot{\phi} = \frac{dV}{d\phi} \tag{2.3}$$

$$\dot{a}^2 = 1 + \frac{a^2}{(D-1)(D-2)}\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right) \tag{2.4}$$

with boundary conditions,

$$\dot{a} = 1\ (t=0), \qquad \dot{a} = -1\ (t = t_1), \qquad \dot{\phi} = 0\ (t = 0, t = t_1) \tag{2.5}$$

where the dot implies differentiation with respect to t. These boundary conditions correspond to the situation where we have ϕ settled safely at their minima for coordinates, $t = 0, t_1$. We are particularly interested in the thin wall limit, where we may approximate, $\phi = \phi_-$ for $t < t_0$ and $\phi = \phi_+$ for $t > t_0$. This would correspond to a 'bubble' with different cosmological constants on either side. The metric would yield as that of a maximally symmetric space with given cosmological constants. Since we have assumed that $V(\phi_+) > V(\phi_-) = 0$, we would have a flat space in the region $t < t_0$ and (Euclidean) de Sitter space in $t > t_0$.

Before turning back to Lorentzian signature, we wish to convert to conformal coordinates, that is, coordinates such that,

$$dX = dt/a(t) \tag{2.6}$$

Then we may write the metric as,

$$ds^2 = a^2(X)(dX^2 + d\theta^2 + \sin^2\theta d\Omega_{D-2}^2) \tag{2.7}$$

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The metric for the CDL instanton can be written in this coordinate system as,

$$a(X) = \begin{cases} \frac{e^{X-X_0}}{\cosh X_0} & (X < X_0) \\ \frac{1}{\cosh X} & (X > X_0) \end{cases} \tag{2.8}$$

The analytic continuation required to obtain the timelike flat region is,

$$X = T + i\pi/2, \qquad ia(T) = a(T + i\pi/2), \qquad \theta \to iR \; (\mu \to il) \tag{2.9}$$

which sends slices of $(D-1)$-spheres to slices of $(D-1)$-hyperbolic spaces. The μ is the geodesic distance on the sphere, where l is the geodesic distance on the hyperbolic space. This yields the metric,

$$ds^2 = a(T)^2(-dT^2 + dR^2 + \sinh^2 R d\Omega_{D-2}^2) = a(T)^2(-dT^2 + dH_{D-1}^2) \tag{2.10}$$

where now,

$$a(T) = \left(\frac{e^{-X_0}}{\cosh X_0}\right) e^T \tag{2.11}$$

which provides the metric for the timelike region inside the bubble. dH_n^2 denotes the metric for the n-dimensional hyperbolic space. Note that we have a well defined spatial infinity in this region, namely at $R \to \infty$ which is an S^{D-2}. With respect to figure 1, this metric describes region A. The thin curves inside this region denotes constant T slices which are H^{D-1}s. Σ is at $R \to \infty$. We will be obtaining the graviton and scalar propagator between two points in this region and taking it to the boundary Σ.

It will prove convenient to use Poincaré coordinates to describe the hyperbolic slices, in which case we obtain the metric,

$$ds^2 = a(T)^2 \left(-dT^2 + \frac{dz^2 + dx_1^2 + \cdots + dx_{D-2}^2}{z^2}\right) \tag{2.12}$$

In these coordinates, Σ lies at $z \to 0$.

For our purposes we are not interested in the spacelike region of the CDL background, but for the sake of the completeness in the argument, the continuation that yields the metric for this region is,

$$\theta \to it' + \pi/2 \tag{2.13}$$

which results in the metric,

$$ds^2 = a^2(X)(dX^2 - dt'^2 + \cosh^2 t' d\Omega_{D-2}^2) \tag{2.14}$$

This describes region B of figure 1, where the thin curves inside the region denotes constant X slices, and the thick bubble wall is at $X = X_0$.

These two regions are patched together at $T = -\infty$ and $X = -\infty$, which is the thick line in figure 1 that divides region A and B.

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3 The transverse-traceless graviton propagator

3.1 The equation of motion

We first calculate the transverse-traceless tensor propagator on the Euclidean manifold and writing it out in a form that has a natural analytic continuation. After that, we will do the analytic continuation (2.9) to the timelike region inside the bubble of our CDL background and obtain the desired propagator respectively.

Taking the metric for a unit S^{D-1} to be $\tilde{g}_{ij}$, the whole background metric can be written as,

$$g_{\mu\nu} = \begin{pmatrix} a(X)^2 & 0 \\ 0 & a(X)^2\tilde{g}_{ij} \end{pmatrix} \tag{3.1}$$

Also, for convenience, we define,

$$N \equiv \frac{D-2}{2} \tag{3.2}$$

We write the metric as,

$$g_{\mu\nu} + \delta g_{\mu\nu} \tag{3.3}$$

where $g_{\mu\nu}$ is the background metric.

We are interested in the $O(D-1)$, gauge invariant perturbation,[1]

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & a(X)^2 h_{ij} \end{pmatrix} \tag{3.4}$$

where h_{ij} is transverse, traceless on S_{D-1}, that is,

$$\tilde{\nabla}^i h_{ij} = 0, \qquad h^i_i = 0 \tag{3.5}$$

where $\tilde{\nabla}$ is the covariant derivative with respect to the metric $\tilde{g}_{ij}$. (We will use lowercase greek letters to denote coordinates in D dimensions, and use letters from the english alphabets to denote coordinates in its $(D-1)$ slices, be it Euclidean or Lorentzian.) It turns out that,

$$\nabla^\mu \delta g_{\mu\nu} = 0, \qquad \delta g^\mu_\mu = 0 \tag{3.6}$$

where ∇ is the covariant derivative with respect to the metric g_{ij}. Hence $\delta g_{\mu\nu}$ is transverse traceless with respect to g_{ij} also. Defining,

$$\tilde{h}_{ij} = a^{(D-2)/2}(X) h_{ij} \tag{3.7}$$

the relevant part of the action concerning $\tilde{h}_{ij}$ is,

$$S = \frac{1}{2}\int dX d\Omega_{D-1}\sqrt{\tilde{g}}\tilde{h}_{ij}\left[-\partial_X^2 + \mathrm{U}(X) + 2 - \tilde{\Box}\right]\tilde{h}^{ij} \tag{3.8}$$

[1]The argument that these perturbations are gauge invariant are presented in various literature, for example in [13]. We will later point out a subtlety that arises in H^{D-1}, namely that certain modes we have to consider turn out to depend on gauge. We will address these issues in section 4.

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where $\tilde{\Box} = \tilde{\nabla}^i\tilde{\nabla}_i$, $\tilde{g} = \det\tilde{g}_{ij}$ and U is defined as,

$$U \equiv (a^N)''/a^N \tag{3.9}$$

where f' denotes df/dX, and $a(X)$ is given by (2.8).

Hence if we define,

$$\hat{G}^{ij}{}_{i'j'}(X_1, X_2, \Omega_1, \Omega_2) = a^N(X_1)a^N(X_2) < h^{ij}(X_1, \Omega_1)h_{i'j'}(X_2, \Omega_2) > \tag{3.10}$$

this satisfies,

$$\left[-\partial^2_{X_1} + \mathrm{U}(X_1) + 2 - \tilde{\Box}_1\right]\hat{G}^{ij}{}_{i'j'}(X_1, X_2, \Omega_1, \Omega_2) = \frac{1}{\sqrt{\tilde{g}}}\delta(X_1 - X_2)\delta^{ij}{}_{i'j'}(\Omega_1, \Omega_2) \tag{3.11}$$

where $\delta^{ij}{}_{i'j'}(\Omega_1, \Omega_2)$ is the normalized projection operator onto transverse traceless tensors on S^{D-1}. The subscript 1 implies differentiation with respect to the coordinates, (X_1, Ω_1). It's worth reminding ourselves again that we are working on a Euclidean manifold.

3.2 Decomposition

Due to the $O(D-1)$ symmetry, the Green's function $G^{ij}{}_{i'j'}$ has to respect all the isometries of the $(D-1)$ sphere with respect to the S^{D-1} coordinates of the two points involved, i.e. it should be a maximally symmetric bitensor with respect to its S^{D-1} coordinates. Therefore it should be possible the write it as,

$$\hat{G}^{ij}{}_{i'j'}(X, X', \mu) \tag{3.12}$$

where $\mu(\Omega_1, \Omega_2)$ is the geodesic distance between the two points Ω_1, Ω_2 (see [14] for further discussion).

The solution for the equation (3.11) can be written as,

$$\hat{G}^{ij}{}_{i'j'}(X, X', \mu) = \sum_{p=(N+2)i}^{+i\infty} G_p(X, X')W^{ij}_{(p)i'j'}(\mu) \tag{3.13}$$

We will define G_p and $W^{ij}_{(p)i'j'}(\mu)$, and explain the range of the sum soon.

$W^{ij}_{(p)i'j'}(\mu)$ is a maximally symmetric bitensor on S^{D-1} defined by,

$$W^{ij}_{(p)i'j'}(\mu) = \sum_u q^{(pu)ij\dagger}(\Omega)q^{(pu)}_{i'j'}(\Omega') \tag{3.14}$$

where $q^{(pu)ij}$ are transeverse traceless eigenmodes of

$$\tilde{\Box}q^{(pu)ij} = (N^2 + 2 + p^2)q^{(pu)ij} \tag{3.15}$$

where u denotes all the quantum numbers other than p needed to specify the mode q. These modes are normalized so that

$$\int d^{D-1}x\sqrt{\tilde{g}}q^{(pu)ij}q^{(p'u')*}_{ij} = \delta^{pp'}\delta^{uu'} \tag{3.16}$$

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Note that by definition $W^{ij}_{(p)i'j'}(\mu)$ satisfies,

$$\widetilde{\Box} W^{ij}_{(p)i'j'}(\mu) = (N^2 + 2 + p^2) W^{ij}_{(p)i'j'}(\mu) \tag{3.17}$$

and is transverse(for all the indices) and traceless(for each pair ij and $i'j'$) with respect to $\tilde{g}_{ij}$.

Also, on S^{D-1}, we get eigenmodes that are regular on the whole sphere only for the p values, $p = (N+2)i, (N+3)i, \ldots$ (see [15]) so by completeness of the basis,

$$\sum_{p=(N+2)i}^{+i\infty} W^{ij}_{(p)i'j'}(\mu(\Omega, \Omega')) = \delta^{ij}{}_{i'j'}(\Omega, \Omega')/\sqrt{\tilde{g}} \tag{3.18}$$

We define G_p to be the X, X' dependent function that satisfies,

$$\left[-\partial_X^2 + \mathrm{U}(X) - (p^2 + N^2)\right] G_p(X, X') = \delta(X - X') \tag{3.19}$$

From equations (3.17), (3.18), and (3.19), we see that indeed (3.13) solves (3.11).

3.3 $G_p(X, X')$

In order to obtain $G_p(X, X')$ satisfying (3.19), let's first think about $F_k(X)$ which satisfy

$$\left[-\frac{d^2}{dX^2} + \mathrm{U}(X)\right] F_k(X) = (k^2 + N^2) F_k(X) \tag{3.20}$$

for a uniform background without any kind of wall. We think about the cases when, $a \propto (1/\cosh X), e^X$ each corresponding to the dS, and flat background.

Then, defining,

$$W \equiv \ln(a^N), \qquad w \equiv \ln a \tag{3.21}$$

we get,

$$U(X) = W'^2 + W'' = (N+1)Nw'^2 - N \tag{3.22}$$
$$\tilde{U}(X) = W'^2 - W'' = N(N-1)w'^2 + N \tag{3.23}$$

where we've used the property,

$$w'^2 - w'' = 1 \tag{3.24}$$

The equation,

$$\left[-\frac{d^2}{dX^2} + N(N+1)w'^2 - N\right] F_k = (N^2 + k^2) F_k \tag{3.25}$$

can be solved in terms of the hypergeometric function $F(a, b; c; z)$ by,

$$F_{k,dS} \equiv e^{ikX} F(-N, N+1; 1 - ik; (1 - \tanh X)/2) \tag{3.26}$$

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for dS, and

$$F_{k,Flat} \equiv e^{ikX} \tag{3.27}$$

for flat space, where we took the boundary conditions to be,

$$F_{k,dS} \to e^{ikX} \qquad X \to \infty \tag{3.28}$$

$$F_{k,Flat} \to e^{ikX} \qquad X \to -\infty \tag{3.29}$$

Now let's introduce the wall. If we have different w' for $X > X_0$ and $X < X_0$, we get,

$$w'^2 - w'' = 1 + A_0 \delta(X - X_0) \tag{3.30}$$

for,

$$A_0 = e^{X_0} / \cosh X_0 \tag{3.31}$$

Hence if we define $A \equiv -N A_0$, the Schrödinger equation,

$$\left[-\frac{d^2}{dX^2} + \mathrm{U}(X) \right] u_k = E_k u_k \tag{3.32}$$

for $\mathrm{U}(X)$ defined by (3.9) for (2.8) becomes,

$$\left[-\frac{d^2}{dX^2} + N(N+1)w' - N + A\delta(X - X_0) \right] u_k = E_k u_k \tag{3.33}$$

where $w = -\ln(\cosh X)$ for $X > X_0$ and $w = X + (\text{constant})$ for $X < X_0$.

Since we already know the eigenfunctions in the separate domains $X > X_0$ and $X < X_0$, the equation can be solved by finding how these waves scatter off the domain wall. For unbounded states, we may write,

$$u_{1k} = \begin{cases} F_{k,L} + \mathbb{R} F_{-k,L} & (X < X_0) \\ \mathbb{T} F_{k,R} & (X > X_0) \end{cases} \tag{3.34}$$

$$u_{2(-k)} = \begin{cases} \mathbb{T}_r F_{-k,L} & (X < X_0) \\ F_{-k,R} + \mathbb{R}_r F_{k,R} & (X > X_0) \end{cases} \tag{3.35}$$

for $E_k = k^2 + N^2$ where,

$$F_{k,L} = F_{k,Flat}, \qquad F_{k,R} = F_{k,dS} \tag{3.36}$$

and $\mathbb{R}, \mathbb{T}, \mathbb{R}_r, \mathbb{T}_r$ are scattering coefficients which depend on k.

Solving the boundary conditions to obtain the reflection coefficient $\mathbb{R}$, we obtain,

$$\mathbb{R} = -e^{2ikX_0} \frac{[F'_{k,R}(X_0) - AF_{k,R}(X_0)] - ikF_{k,R}(X_0)}{[F'_{k,R}(X_0) - AF_{k,R}(X_0)] + ikF_{k,R}(X_0)} = e^{2ikX_0} \mathcal{R} \tag{3.37}$$

where $\mathcal{R}$ can be written in terms of hypergeometric functions as,

$$\mathcal{R} = -\frac{N(1-t)F(-N+1, N+1; 1-ik; t)}{(ik+N)F(-N, N; 1-ik; t)}, \qquad t = \frac{e^{-X_0}}{2\cosh X_0} \tag{3.38}$$

We note the following properties of $\mathcal{R}$:

1. The poles ia_n of $\mathcal{R}$ in the upper half plane correspond to bound states. They are purely imaginary, and $a_n \leq N$.

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2. Regardless of the value of X_0, $k = iN$ is always a pole of $\mathcal{R}$.

When N is an integer, $\mathcal{R}$ has the following properties.

1. $\mathcal{R}$ is a rational function with respect to k.

2. $\mathcal{R}$ has N other poles, which are pure imaginary and lie between $(-iN, iN)$.

3. In the limit $X_0 \to -\infty$, the poles other than iN tend to $0, i, \ldots, i(N-1)$.

4. In the limit $X_0 \to \infty$, the poles other than iN tend to $-i, -2i, \ldots, -iN$.

5. $k = -iN$ is always a zero of $\mathcal{R}$.

When N is a half integer, $\mathcal{R}$ exhibits some very interesting properties. As in the integer case, all poles lie on the imaginary axis below $k = iN$, and has only a finite number of poles in $[-iN, iN]$, but in the limit $k \to -i\infty$ the pole structure varies starkly:

1. For $X_0 \geq 0$, $\mathcal{R}(k)$ has an infinite number of poles on the imaginary axis of the lower half plane. (Appendix A)

2. For $X_0 < 0$, $\mathcal{R}(k)$ has only a finite number of poles on the imaginary axis of the lower half plane, but has an infinite number of complex poles on the lower half plane for $-\epsilon < X_0 < 0$ for some $\epsilon > 0$. (Appendix A)

Now we are ready to solve (3.19).

$$G_p(X, X') = \frac{1}{\Delta_p}[\Psi_p^r(X)\Psi_p^l(X')\Theta(X - X') + \Psi_p^l(X)\Psi_p^r(X')\Theta(X' - X)] \tag{3.39}$$

where $\Psi_p^r(X)$ is the solution to the Schrödinger equation (3.33), that goes to e^{ipX} as $X \to \infty$, and $\Psi_p^l(X)$ the solution that goes to e^{-ipX} as $X \to -\infty$. Δ_p is defined to be the Wronskian of Ψ_p^r and Ψ_p^l.

Since we can write $\Psi_p^r(X)$, $\Psi_p^l(X)$ in terms of u_k, namely,

$$\Psi_p^r(X) = u_{1p}(X), \qquad \Psi_p^l(X) = u_{2(-p)}(X) \tag{3.40}$$

and for the flat side of the bubble, we get,

$$G_p(X, X') = \frac{i}{2p}\left(e^{ip\delta X} + \mathbb{R}(p)e^{-ip\bar{X}}\right) \qquad (X, X' < X_0) \tag{3.41}$$

where

$$\delta X = \begin{cases} X - X' & (X > X') \\ X' - X & (X' > X) \end{cases} \tag{3.42}$$

$$\bar{X} = X + X' \tag{3.43}$$

and the reflection coefficient is given by (3.37).

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3.4 $W^{ij}_{(p)i'j'}(\mu)$

The calculation of the maximally symmetric bitensor $W^{ij}_{(p)i'j'}(\mu)$ in this section is carried out follwing the steps of [16].

A maximally symmetric bitensor in general can be written as,

$$\begin{aligned}T_{iji'j'} =& t_1 g_{ij} g_{i'j'} + t_2[n_i g_{ji'} n_{j'} + n_j g_{ii'} n_{j'} + n_i g_{jj'} n_{i'} + n_j g_{ij'} n_{i'}] \\ &+ t_3[g_{ii'} g_{jj'} + g_{ji'} g_{ij'}] + t_4 n_i n_j n_{i'} n_{j'} + t_5[g_{ij} n_{i'} n_{j'} + n_i n_j g_{i'j'}]\end{aligned} \tag{3.44}$$

where t_i are functions of μ, the length of the geodesic that connects Ω and Ω'. Here, $n_i(\Omega, \Omega')$, $n'_i(\Omega, \Omega')$ are unit vectors each at Ω and Ω' pointing away from Ω' and Ω respectively. $g_i^{j'}$ is the parallel propagator along the geodesic.

By using the tracelessness of $W^{ij}_{(p)i'j'}(\mu)$, we can write,

$$W^{ij}_{(p)i'j'}(\mu) = Q_p w^I(\alpha_p(z)) t_{I\ \ i'j'}^{ij}|_{z=\cos^2(\frac{\mu}{2})} \tag{3.45}$$

where we have defined

$$z \equiv \cos^2\left(\frac{\mu}{2}\right) \tag{3.46}$$

and

$$t_{1\ \ i'j'}^{ij} = [g_{ij} - (D-1)n_i n_j][g_{i'j'} - (D-1)n_{i'} n_{j'}] \tag{3.47}$$

$$t_{2\ \ i'j'}^{ij} = 4n_{(i} g_{j)(i'} n_{j')} + 4n_i n_j n_{i'} n_{j'} \tag{3.48}$$

$$t_{3\ \ i'j'}^{ij} = g_{ii'} g_{jj'} + g_{ji'} g_{ij'} - 2g_{ij} n_{i'} n_{j'} - 2n_i n_j g_{i'j'} - 2(D-1) n_i n_j n_{i'} n_{j'} \tag{3.49}$$

$$t^{ij}_{\ \ i'j'} = t_{1\ \ i'j'}^{ij} - N t_{2\ \ i'j'}^{ij} - N t_{3\ \ i'j'}^{ij} \tag{3.50}$$

We refer the reader to appendix B for the explicit expression for $w^I(\alpha_p(z))$ and Q_p. We also have defined $t^{ij}_{\ \ i'j'}$ which will come in handy later.

We first obtain $w^I(\alpha_p(z))$ starting from equation (3.45) up to a constant by imposing transverseness and the condition (3.17). The result is given by equation (B.2).

The normalization constant Q_p given by equation (B.4) is obtained by considering the degeneracy of the modes $q_{ij}^{(pu)}$. More specifically, this is done by contracting $i'j'$ and ij and taking $\Omega = \Omega'$ in equation (3.14) and integrating over the whole sphere by Ω. By doing this, from (3.45), the r.h.s. of the contracted and integrated (3.14) would yield some numerical constant(which can be obtained from (B.2)) times Q_p times the volume of the $(D-1)$ sphere. The l.h.s. of the contracted and integrated (3.14) would yield the degeneracy of the modes $q_{ij}^{(pu)}$ with given p, due to equation (3.16).

Note that in order for α_p and β_p defined by (B.1), (B.3) to be well defined, and hence $W^{ij}_{(p)i'j'}(\mu)$ to be well defined on the whole sphere (for all $0 \leq z \leq 1$) p must have the values $p = i(N+2), i(N+3), \ldots$.

3.5 Analytic continuation

Since we have obtained $G_p(X, X')$ and $W^{ij}_{(p)i'j'}$ showing up in equation (3.13) in the previous two sections, it is straight forward to write down the hatted propagator for the instanton.

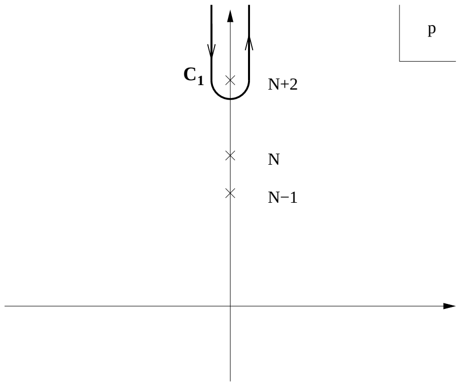

Figure 2. The contour C_1.

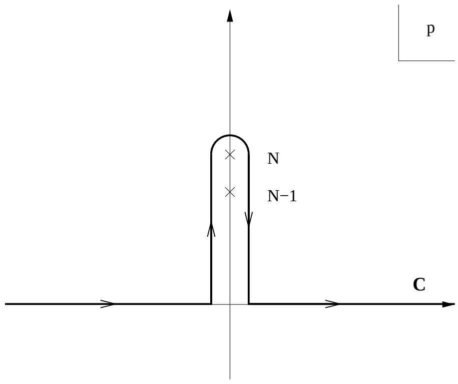

Figure 3. The contour C.

The problem is that we want to analytically continue this to the time-like Lorentzian region of our background (namely to carry out equation (2.9)) but this is not a trivial thing to do.

The problem is that we want to think about the propagator as we take the points concerned to the boundary sitting at spacelike infinity(l: large) of this region. But by plugging in (2.9) to (3.13) we don't get a convergent sum in this limit. This is because

$$W^{ij}_{(i(N+2+n))i'j'}(il) \sim e^{(2+n)l} t^{ij}{}_{i'j'} \tag{3.51}$$

for large l, as can be easily verified by the asymptotic limit of hypergeometric functions.

In order to achieve this objective, we must employ a more sophisticated method previously utilized by various authors [5, 11–13]. The way do this is by expressing the sum (3.13) as,

$$\hat{G}^{ij}{}_{i'j'}(X, X', \mu) = \int_{C_1} \frac{dp}{2\pi i} \frac{\Gamma(-ip - N - 1)\Gamma(ip + N + 2)}{(-1)^{-ip-N-2}} \tag{3.52}$$

$$\times G_p(X, X') W^{ij}_{(p)i'j'}(\mu) \tag{3.53}$$

where the contour C_1 is defined to be one that comes down from $i\infty$ on the left side of the imaginary axis of the complex p plane, and pivots around $p = i(N+2)$ to go back to $i\infty$ by the right side of the imaginary axis. The Γ functions pick out the appropriate poles with the desired residues. This is depicted in figure 2.

Plugging in (3.41) to the above expression, we obtain,

$$\begin{aligned}\hat{G}^{ij}{}_{i'j'}(X, X', \mu) = \int_{C_1} \frac{dp}{4\pi p} \frac{\Gamma\left(-ip - N - 1\right)\Gamma(ip + N + 2)}{(-1)^{-ip-N-2}} \\ \times \left(e^{ip\delta X} + \mathbb{R}(p) e^{-ip\bar{X}}\right) W^{ij}_{(p)i'j'}(\mu)\end{aligned} \tag{3.54}$$

The first term yields the Green's function for a flat background. Let's focus our attention to the second term, which we denote by $\hat{G}^{ij}{}_{i'j'}^{\bar{X}}$.

Now this contour can be safely deformed to the contour C, which we define to run along the real axis of the p plane, with a 'jump' over $p = iN$. This is depicted in figure 3. The contour deformation is justified by the following reasons.

First, by writing the previous equation as,

$$\hat{G}^{ij}{}_{i'j'}^{\bar{X}}(X,X',\mu) = \int_{C_1} dp \left(\frac{i}{2p}\right) \frac{e^{ip(2X_0-\bar{X})}}{1-e^{2\pi(p-iN)}} \mathcal{R}(p) W^{ij}_{(p)i'j'}(\mu) \tag{3.55}$$

Since $\bar{X} - 2X_0 < 0$, the first piece in the integrand decays exponentially at infinity on the upper half plane, as long as the contour does not pass through $p = in$. We also know that $F(a,b;c;z) \approx 1 + \mathcal{O}(1/|c|)$ for $c \to \infty$ so $\mathcal{R}(p)$ behaves nicely in this region.

Also, we note that α_p can be written as,

$$\begin{aligned}\alpha_p(z) &= F(N+2+ip, N+2-ip; N+5/2; z) \\ &= \Gamma(N+5/2)(z-z^2)^{3/2-N} P^{3/2-N}_{ip-1/2}(1-2z) \\ &\sim \left(\frac{1}{-ip}\right)^{N-1}\end{aligned} \tag{3.56}$$

for $p \to i\infty$ and hence $W^{ij}_{(p)i'j'}(\mu)$ also behaves nicely.

Finally, there aren't any poles in the integrand between iN and $i(N+2)$ on the imaginary axis, (since by (B.4), $W^{ij}_{(p)i'j'}(\mu) = 0$ at $p = i(N+1)$) so we may carry out the deformation as we please. Hence,

$$\hat{G}^{ij}{}_{i'j'}^{\bar{X}}(X,X',\mu) = \int_C \frac{dp}{4\pi p} \frac{\Gamma(-ip-N-1)\Gamma(ip+N+2)}{(-1)^{-ip-N-2}} \tag{3.57}$$

$$\times \mathbb{R}(p) e^{-ip\bar{X}} W^{ij}_{(p)i'j'}(\mu) \tag{3.58}$$

Now let's do the analytic continuation,

$$X = T + i\frac{\pi}{2}, \qquad \mu = il \tag{3.59}$$

Then after pulling out all the trivial constants out in front and sorting out the terms, the analitically continued propagator piece $\hat{G}^{ij}{}_{i'j'}^{\bar{T}}(T,T',l)$ can finally be written as,

$$\begin{aligned}\hat{G}^{ij}{}_{i'j'}^{\bar{T}}(T,T',l) = C_0 \int_C dp \mathbb{R} e^{-ip\bar{T}} Y^{ij}_{(p)i'j'}(il) \\ \times (p^2 + (N+1)^2)\Gamma(ip+N-1)\Gamma(-ip+N-1)\end{aligned} \tag{3.60}$$

where we have conveniently defined,

$$Y^{ij}_{(p)i'j'}(il) \equiv \frac{1}{Q_p} W^{ij}_{(p)i'j'}(il) = w^I(\alpha_p(z)) t^{ij}_{I\ i'j'}|_{z=\cosh^2 \frac{l}{2}} \tag{3.61}$$

3.6 The large l limit

In this section, we will write out the 'holographic expansion' for the graviton propagator, i.e. in a form convenient to examine its potential holographic duality. In order to do this, it is convenient to invoke the 'generalized Green function's we have defined in appendix C.

We first define,

$$a_p(z) = \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip} F\left(\frac{D+2}{2} - ip, \frac{1}{2} - ip; 1-2ip; \frac{1}{z}\right) \tag{3.62}$$

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and then define $G^{ij}_{H\ i'j'}$ to be,

$$G^{ij}_{H\ i'j'}(l,\Delta) = w^I(a_{i(\Delta-N)})t^{ij}_{I\ i'j'}|_{z=\cosh^2\frac{l}{2}} \tag{3.63}$$

At large l, (or for the Poincaré coordinates in H^{D-1}, small z) this behaves as,

$$\begin{aligned} G^{ij}_{H\ i'j'}(l,\Delta) &\sim C(\Delta-2N)(\Delta-2N+1)e^{-\Delta l}t^{ij}_{\ \ i'j'} + \mathcal{O}\left(e^{-(\Delta+2)l}\right) \\ &\sim C(\Delta-2N)(\Delta-2N+1)\frac{z^\Delta z'^\Delta}{|x-x'|^{2\Delta}}t^{ij}_{\ \ i'j'} + \mathcal{O}\left(\frac{z^{\Delta+2}z'^{\Delta+2}}{|x-x'|^{2\Delta+4}}\right) \end{aligned} \tag{3.64}$$

The Δ dependence of the coefficient of the leading order behavior will prove to be important.[2] Also,

$$G^{ij}_{H\ i'j'}(l,\Delta) \propto G^{ij}_{M\ i'J'}(l,\Delta(\Delta-2N)) \tag{3.65}$$

for $\Delta > N$, $\Delta \neq 2N$, where $G^{ij}_{M\ i'J'}(l,m)$ is the massive transverse traceless propagator on H^{D-1} with mass m. We know from AdS/CFT that this corresponds to a two point function for a dimension Δ traceless tensor of the boundary theory of the $EAdS_{D-1}$ [17].

Due to the identity between hypergeometric functions,

$$\alpha_p(z) = \frac{\Gamma\left(N+\frac{5}{2}\right)\Gamma(-2ip)}{\Gamma(N+2-ip)\Gamma(\frac{1}{2}-ip)}a_{-p}(z) + \frac{\Gamma(N+\frac{5}{2})\Gamma(2ip)}{\Gamma(N+2+ip)\Gamma(\frac{1}{2}+ip)}a_p(z) \tag{3.66}$$

so using the linearity of w^I, we may write (3.60) as,

$$\begin{aligned} \hat{G}^{ij\ \bar{T}}_{\ \ i'j'} = C_0\int_C dp\mathbb{R}e^{-ip\bar{T}}\Bigg[&\frac{\Gamma(-ip)\Gamma(ip+N-1)(N+1+ip)}{2^{-2ip-1/2}(N-ip)(N-1-ip)}w^I(a_{-p})t^{ij}_{I\ i'j'} \\ &+\frac{\Gamma(ip)\Gamma(-ip+N-1)(N+1-ip)}{2^{2ip-1/2}(N+ip)(N-1+ip)}w^I(a_p)t^{ij}_{I\ i'j'}\Bigg] \end{aligned} \tag{3.67}$$

where we have absorbed some overall factors into C_0. We have used the fact that,

$$w^I(\alpha_p(z)) = w^I(c_1a_{ip}(z)+c_2a_{-ip}(z)) = c_1w^I(a_{ip}(z)) + c_2w^I(a_{-ip}(z)) \tag{3.68}$$

This can be re-written as,

$$\begin{aligned} \hat{G}^{ij\ \bar{T}}_{\ \ i'j'} = C_0\int_C dp\mathbb{R}e^{-ip\bar{T}}\Bigg[&\frac{\Gamma(-ip)\Gamma(ip+N-1)(N+1+ip)}{2^{-2ip-1/2}(N-ip)(N-1-ip)}G^{ij}_{H\ i'j'}(l,N+ip) \\ &+\frac{\Gamma(ip)\Gamma(-ip+N-1)(N+1-ip)}{2^{2ip-1/2}(N+ip)(N-1+ip)}G^{ij}_{H\ i'j'}(l,N-ip)\Bigg] \end{aligned} \tag{3.69}$$

by (3.64). In the large l limit,

$$G^{ij}_{H\ i'j'}(l,N\pm ip) \sim e^{-(N\pm ip)l}t^{ij}_{\ \ i'j'} \tag{3.70}$$

Hence in this limit, C for the former term of equation (3.69) may be deformed downward while the latter term may be deformed upward. This is because the asymptotic behavior

[2] Thanks to Leonard Susskind and Yasuhiro Sekino in helping me realize this.

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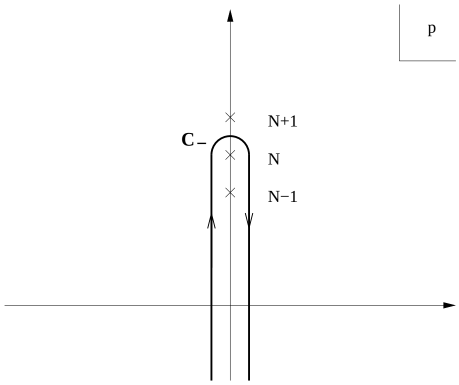

Figure 4. The contour C_-.

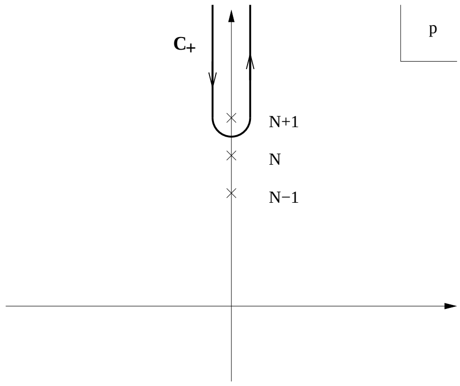

Figure 5. The contour C_+.

of all the other components in the product is at worst $\sim e^{ap}$ at $|p| \to \infty$ on the half-plane concerned for some fixed number a.

Define the contour C_- to be the contour coming from $-i\infty$ on the left side of the imaginary axis, pivoting around $p = iN$ and going back down to $-i\infty$ on the right side of the imaginary axis. Define the contour C_+ to be the contour coming from $i\infty$ on the left side of the imaginary axis, pivoting around just above $p = iN$) and going back up to $i\infty$ on the right side of the imaginary axis. These are depicted in figure 4 and figure 5 respectively.

Now we may write,

$$\begin{aligned} G^{ij\ \bar{T}}_{\ \ i'j'} &= C_0 \int_{C_-} dp \mathbb{R} e^{(-N-ip)\bar{T}} \frac{\Gamma(-ip)\Gamma(ip+N-1)(N+1+ip)}{2^{-2ip-1/2}(N-ip)(N-1-ip)} G^{ij}_{Hi'j'}(l,N+ip) \\ &\quad + C_0 \int_{C_+} dp \mathbb{R} e^{(-N-ip)\bar{T}} \frac{\Gamma(ip)\Gamma(-ip+N-1)(N+1-ip)}{2^{2ip-1/2}(N+ip)(N-1+ip)} G^{ij}_{Hi'j'}(l,N-ip) \\ &\equiv I_- + I_+ \end{aligned} \tag{3.71}$$

Note that we have gotten rid of the hat on the propagator by multiplying it by $e^{-N\bar{T}}$.

The poles of the integrand of I_+ are given as the following.

1. $p = in$ for integers n.
2. $p = iN$, $p = i(N-1)$
3. $p = -i(N-1+n)$ for non-negative integer n other than $n = 2$.
4. The poles of $\mathcal{R}$ (including $p = iN$).

The non-negative integer poles come from the gamma function while the negative integer poles come from the poles of $G^{ij}_{H\ i'j'}(l, N-ip)$. Note that these poles may 'pile up.' For example, when N is an integer, the pole $p = iN$ becomes a triple pole due to the $p = in$ pole of the first line, the $p = iN$ pole of the second line, and the $p = iN$ pole that comes from the reflection coefficient. Note that this is written for the general case. For special values of X_0 the zeros coming from $\mathcal{R}$ may cancel some poles mentioned above. For reasons evident later, we mention the behavior of the integrand of I_+ at $p = iN$ and $i(N-1)$.

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1. $p = iN$ is a triple(double) pole for integer(half-integer) N.

2. $p = i(N-1)$ is a double(single) pole for integer(half-integer) N.

The poles of the integrand of I_- are given as the following.

1. $p = in$ for integers n.

2. $p = -iN$, $p = -i(N-1)$

3. $p = i(N-1+n)$ for non-negative integer n other than $n = 2$.

4. The poles of $\mathcal{R}$ (including $p = iN$).

The non-positive integer poles come from the gamma function while the positive integer poles come from the poles of $G^{ij}_{H\ i'j'}(l, N+ip)$. The features discussed about the latter piece apply to this piece as well. One notable feature in this case is that $p = -iN$ always turns out to be a simple pole. To elaborate, for integer N, we get the contributions of the first line and second line to get a double pole at $-iN$ while a zero at $-iN$ for $\mathcal{R}$ appears to make the pole simple. This zero in $\mathcal{R}$ doesn't exist for half-integer N, making the pole simple also in this case. The behavior of the integrand of I_- at $p = \pm iN$ and $\pm i(N-1)$ are as the following.

1. $p = iN$ is a triple(double) pole for integer(half-integer) N.

2. $p = -iN$ is always a single pole.

3. $p = i(N-1)$ is a double(single) pole for integer(half-integer) N.

4. $p = -i(N-1)$ is a double(single) pole for integer(half-integer) N.

I_+ can be written easily as we don't have to deal with any double poles.

$$I_+ = \sum_{n=[N]+1}^{\infty} A_n e^{(-N+n)\bar{T}} G^{ij}_{H\ i'j'}(l, N+n) \tag{3.72}$$

I_- has some double poles we have to think about. The simple pole contribution can be written as,

$$\begin{aligned}
I_{-,1} = & \sum_{n=1}^{[N]} A_n e^{(-N+n)\bar{T}} G^{ij}_{H\ i'j'}(l, N+n) \\
& + \sum_{n=-\infty}^{0} B_n e^{(-N+n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-n) \\
& + \sum_{ia_n:\ \text{poles of } \mathbb{R};\ a_n<N} C_n e^{(-N+a_n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-a_n) \\
& + \delta_{N,[N]+1/2}\big(B_N G^{ij}_{H\ i'j'}(l,0) + B_{(N-1)} e^{-\bar{T}} G^{ij}_{H\ i'j'}(l,1) \\
& + B_{-(N-1)} e^{-(2N-1)\bar{T}} G^{ij}_{H\ i'j'}(l, 2N-1) + B_{-N} e^{-2N\bar{T}} G^{ij}_{H\ i'j'}(l, 2N)\big)
\end{aligned} \tag{3.73}$$

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For odd dimensions, we always get the double pole at $p = iN$;

$$I_{-,2,odd} = J_N \bar{T} G^{ij}_{H\ \ i'j'}(l,0) + K_N \frac{\partial}{\partial\Delta} G^{ij}_{H\ \ i'j'}(l,\Delta)|_{\Delta=0} \tag{3.74}$$

Note that,

$$\frac{\partial}{\partial\Delta} G^{ij}_{H\ \ i'j'}(l,\Delta)|_{\Delta=\Delta_0} \sim l e^{-\Delta_0 l} t^{ij}_{\ \ i'j'} \quad \text{for large } l \tag{3.75}$$

For even dimensions, we get double poles at $p = \pm i(N-1)$ and a triple pole at $p = iN$. The double poles give rise to the terms,

$$\begin{aligned} I_{-,2,even} = \; & D_{N-1} \bar{T} e^{-\bar{T}} G^{ij}_{H\ \ i'j'}(l,2N-1) + B^H_{N-1} e^{-\bar{T}} H^{ij}_{0\ \ i'j'}(l,1) \\ & + J_{-(N-1)} \bar{T} e^{-(2N-1)\bar{T}} G^{ij}_{H\ \ i'j'}(l,2N-1) \\ & + K_{-(N-1)} e^{-(2N-1)\bar{T}} \frac{\partial}{\partial\Delta} G^{ij}_{H\ \ i'j'}(l,\Delta)|_{\Delta=2N-1} \end{aligned} \tag{3.76}$$

and the triple pole gives rise to the term,

$$\begin{aligned} I_{-,3,even} = \; & D_N \bar{T} G^{ij}_{H\ \ i'j'}(l,2N) + F_N \bar{T}^2 G^{ij}_{H\ \ i'j'}(l,2N) \\ & + B^H_N H^{ij}_{0\ \ i'j'}(l,0) + J^H_N \bar{T} H^{ij}_{0\ \ i'j'}(l,0) + K^H_N H^{ij}_{1\ \ i'j'}(l,0) \end{aligned} \tag{3.77}$$

Note that in the even dimensional case we have neglected the pieces already put into $I_{-,1}$. The definition for the functions H_0 and H_1 are given in appendix E, by equations (E.9) and (E.10).

We see that in both the odd and even dimensional case, the asymptotic behavior of the propagator is logarithmic, that is that it behaves as $\sim l t^{ij}_{\ \ i'j'}$.

4 Gauge choice

In the previous section, we have obtained the expression for the transverse traceless graviton propagator. As previously mentioned at the beginning of section 3, the transverse traceless perturbation of the graviton is 'almost' gauge invariant, that is, the transeverse tracelessness fixes the gauge degrees of freedom except with respect to a few modes.

In section 4.1 we will elaborate on what we mean by saying that there exists residual gauge freedom. In this section we will also present a 'naive' way of getting rid of those gauge degrees of freedom. We will present the propagator that is gauge-fixed in this manner in 4.2.

Finally, in section 4.3 we will discuss the subtlety overlooked in the gauge-fixing method presented in the first subsection and present what we believe is to be the correct gauge-fixed propagator.

4.1 Gauge choice and contour integration

An important issue we must address is the residual gauge degrees of freedom we haven't gotten rid of in calculating the graviton correlator. In other words, we have to get rid of "degenerate modes" of the transverse-traceless graviton.

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In order to clarify what 'degenerate' means, we first decompose the graviton in our background. We know that the (perturbation of the) graviton on a H^{D-1} slice of the bubble can be (almost) uniquely decomposed as,

$$\delta g_{ij} = \frac{1}{D-1} h\tilde{\gamma}_{ij} + 2\left(\tilde{\nabla}_i \tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box}\right) E + 2F_{(i|j)} + h_{ij} \tag{4.1}$$

Where $\tilde{\gamma}_{ij}$ is the unit H^{D-1} metric (see for example, [13]). We have used $|j$ as a shorthand for $\tilde{\nabla}_j$. Here h, E are scalars, F_i is a transverse vector, and as we know, h_{ij} is a transverse traceless symmetric tensor. We have stated that we are only interested in the two point function of the h_{ij} perturbation.

Hence the path integral we carry out concerns modes of the transverse traceless perturbation on H^{D-1}. Note that,

$$\Psi^h_{p'}(T + i\pi/2) r^{(pu)ij}(\mathcal{H}) \tag{4.2}$$

would serve as an orthonormal basis of such modes, where $r^{(pu)ij}$ are transeverse traceless eigenmodes of

$$\tilde{\Box} r^{(pu)ij} = -(N^2 + 2 + p^2) r^{(pu)}_{i'j'} \tag{4.3}$$

which are normalized so that

$$\int d^{D-1}x \sqrt{\tilde{\gamma}} r^{(pu)ij} r^{(p'u')\dagger}_{ij} = \delta(p - p')\delta^{uu'} \tag{4.4}$$

in H^{D-1}. Note that $\tilde{\Box}$ is the Laplacian with respect to $\tilde{\gamma}_{ij}$, $\tilde{\gamma} = \det\tilde{\gamma}_{ij}$, and as before, u denotes quantum numbers other than p. $\Psi^h_{p'}(X)$ are defined in (3.40).

The problem is that there are modes that introduce an ambiguity to the decomposition (4.1). Suppose there is a transverse mode $F^{(pu)i}$ such that, $F^{(pu)(i|j)}$ is transverse, traceless and satisfies,

$$\tilde{\Box} F^{(pu)(i|j)} = -(N^2 + 2 + p'^2) F^{(pu)(i|j)} \tag{4.5}$$

Then for this perturbation of the graviton in the angular direction, it is ambiguous whether to put

$$2F^i = f(T) F^{(pu)i} \tag{4.6}$$

or to put,

$$h^{ij} = f(T) F^{(pu)(i|j)} \tag{4.7}$$

where $f(T)$ an arbirary function only of T. The same is true if we had a scalar mode E^{pu} such that, $E_{;ij} - \frac{\tilde{\gamma}_{ij}}{D-1} E^{;i}_{\ ;i}$ is transverse traceless and satisfies,

$$\tilde{\Box}\left(E_{;ij} - \frac{\tilde{\gamma}_{ij}}{D-1} E^{;i}_{\ ;i}\right) = -\left(N^2 + 2 + p''^2\right)\left(E_{;ij} - \frac{\tilde{\gamma}_{ij}}{D-1} E^{;i}_{\ ;i}\right) \tag{4.8}$$

For this perturbation in the angular direction, it is ambiguous whether to put,

$$2E = f(T) E^{(pu)} \tag{4.9}$$

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or to put,

$$h_{ij} = f(T)\left(E_{;ij} - \frac{\tilde{\gamma}_{ij}}{D-1}E^{;i}_{\ ;i}\right) \tag{4.10}$$

This signals a 'degeneracy' in the vector/scalar and tensor modes of the graviton. By 'degenerate modes' we are refering to these modes that may be written in terms of other components in the decomposition (4.1).

The statement we have made in section 3 that we will only consider transverse traceless perturbations is actually a gauge condition; that $E = 0$ and $F^i = 0$. Hence such modes of h^{ij} represent a residual gauge freedom we haven't fixed yet, as these may well be written as perturbations of the scalar/vector modes. Therefore, in order to completely fix the gauge, we should find them and project them out.

We will check in the appendix F that the "supercurvature modes" $p = iN$ and $p = i(N-1)$ are degenerate with vector modes and the scalar mode respectively. Let's see how to project these out from the propagator.

We first start from (3.60). We can write this in a more convenient manner similar to (3.55), which is,

$$G^{ij\ \bar{T}}_{\ \ i'j'}(T, T', l) = \int_C dp\left(\frac{i}{2p}\right)\frac{e^{ip(2X_0 - i\pi - \bar{T})}}{1 - e^{2\pi(p-iN)}}\mathcal{R}(p)W^{ij}_{(p)i'j'}(il) \tag{4.11}$$

In order to see how the individual tensor modes on H^{D-1} contribute to this propagator, we have to go through some steps.

We first define the maximally symmetric bitensor,

$$Z^{ij}_{(p)i'j'}(l) = \sum_u r^{(pu)ij}(\mathcal{H})^\dagger r^{(pu)}_{i'j'}(\mathcal{H}') \tag{4.12}$$

From the general prescription of obtaining maximally symmetric bitensors which come from the sum of well defined modes in S^d and H^d (which is kindly laid out for the case $d = 3$ in [16]) we know that the relation,

$$Z^{ij}_{(p)i'j'}(l) = \frac{Q'_p}{Q_p}W^{ij}_{(p)i'j'}(il) \tag{4.13}$$

holds. This is more explicitly addressed in [15], where a multiple of Q'_p is denoted as a 'spectral function'. From equation (2.107) in this paper, we see that

$$Q'_p = \frac{D[p^2 + (N+1)^2]}{2^{D-1}\pi^{N+1/2}\Gamma(N+1/2)}\frac{\Gamma(ip+N-1)\Gamma(-ip+N-1)}{\Gamma(ip)\Gamma(-ip)} \tag{4.14}$$

The problems is that Q'_p/Q_p turns out to have simple poles for $p = i(N-1), iN$ and $p = i(N+2), i(N+3), \ldots$. (We will only be concerned with the first two poles, as they are the ones relevant to the contour integral.) We must address how to think about the pole of $Z^{ij}_{(p)i'j'}(l)$.

The poles of $Z^{ij}_{(p)i'j'}$ come from the normalization constant of the individual modes that diverge for the given values of p (see [15]). Since $W^{ij}_{(p)i'j'}(il)$ is obtained by multiplying an

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analytic function of p to get rid of the poles in the upperhalf plane, it can be written as,

$$W^{ij}_{(p)i'j'}(il) = \sum_{u:\text{non-zero r'}} r'^{(pu)ij\dagger}(\mathcal{H}) r'^{(pu)}_{i'j'}(\mathcal{H}') \tag{4.15}$$

where $r'^{(pu)ij}$ aren't normalized properly. This means that for certain values of p and u, $r'^{(pu)ij}$ may be zero. This is because symmetric transverse traceless tensor modes on H^{D-1} have different normalization constants for different quantum numbers. For example, the parity even spin 2 tensor modes of have an extra factor of $1/\sqrt{p^2+(N-1)^2}$ in their normalization constant compared to the parity odd spin 2 tensor modes on H^{D-1}.[3] We have modified the sum over u to make this point clear. To state this more clearly, $\{r'^{(pu)ij}\} \subset \{r^{(pu)ij}\}$ and in some cases, $\{r'^{(pu)ij}\} \neq \{r^{(pu)ij}\}$. We also note that,

$$\partial_p W^{ij}_{(p)i'j'}(l) = \sum_{u:\text{non-zero } r'} (\partial_p r'^{(pu)ij\dagger}(\mathcal{H}) r'^{(pu)}_{i'j'}(\mathcal{H}') + r'^{(pu)ij\dagger}(\mathcal{H}) \partial_p r'^{(pu)}_{i'j'}(\mathcal{H}')) \tag{4.16}$$

We stress again that $W^{ij}_{(p)i'j'}(l)$ is well defined(regular) in the upper half plane.

Now we can write (4.11) as,

$$\begin{aligned} G^{ij\ \bar{T}}_{\ \ i'j'}(T,T',l) = & \int_{-\infty}^{\infty} dp \left(\frac{i}{2p}\right) \frac{e^{ip(2X_0 - i\pi - \bar{T})}}{1-e^{2\pi(p-iN)}} \mathcal{R}(p) W^{ij}_{(p)i'j'}(il) \\ & -2\pi i \sum_{\substack{p_R=\{a_n\} \\ \{(N-[N-1/2]),\ldots,N\}}} \mathrm{Res}_{p=ip_R} \left(\frac{i}{2p}\right) \frac{e^{ip(2X_0 - i\pi - \bar{T})}}{1-e^{2\pi(p-iN)}} \mathcal{R}(p) W^{ij}_{(p)i'j'}(il) \end{aligned} \tag{4.17}$$

where ia_n are the positive poles of the reflection coefficient. This can be schematically written as,

$$\begin{aligned} G^{ij\ \bar{T}}_{\ \ i'j'}(T,T',l) = & \sum_{p:\text{real}} \Phi_p(\bar{T}) Z^{ij}_{(p)i'j'}(l) \\ & + \sum_{p=ia_n, i(N-[N-1/2]),\ldots i(N-2)} \Phi_p(\bar{T}) Z^{ij}_{(p)i'j'}(l) \\ & + \Phi_{i(N-1)}(\bar{T}) W^{ij}_{(i(N-1))i'j'}(l) \\ & + \Phi_{iN}(\bar{T}) W^{ij}_{(iN)i'j'}(l) + \Phi'_{iN}(\bar{T}) \partial_p W^{ij}_{(p)i'j'}(l)|_{p=iN} \end{aligned} \tag{4.18}$$

The $\Phi_p(\bar{T})$ denotes the $\bar{T}$ dependence of each component. Note that for certain values of X_0, in and ia_n can coincide to give multiple poles, but this is irrelevant to the point we wish to make now, so we will ignore such subtleties. Note that the last line two lines come from the poles at $p = i(N-1), iN$. From (4.12), (4.15) and (4.16) we see that the expression (4.18) shows explicitly the contribution of each hyperbolic mode to the propagator.

In appendix F, it is shown that indeed the mode sum (4.15) for $p = i(N-1), iN$ can be written as a sum of modes coming from scalar and vector modes. Although the scalar and vector modes might not saturate $\{r^{(pu)ij}\}$ (as we see in the appendix, the scalar mode derivatives only give rise to the even tensor modes), it certainly saturates $\{r'^{(pu)ij}\}$ as some

[3] See section 2 of [15] for more details.

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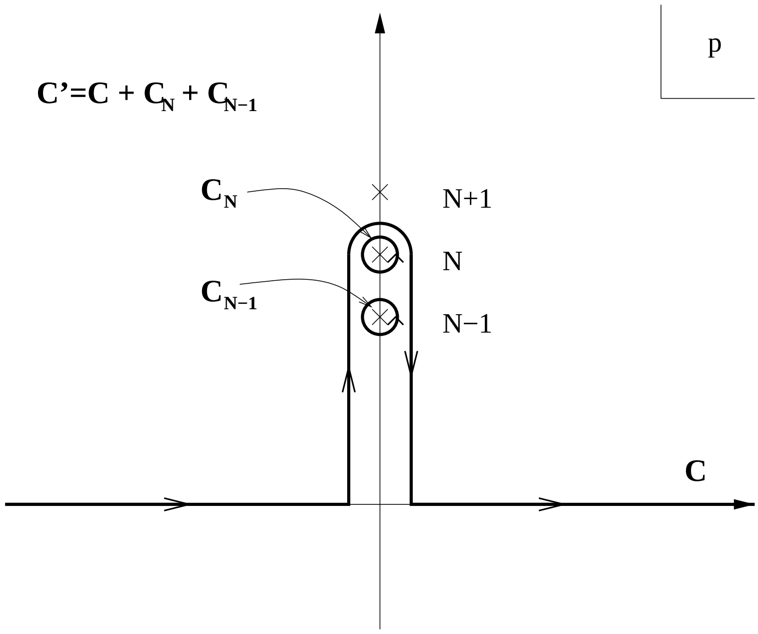

Figure 6. The contour C'.

of the $r^{(pu)ij}$ obtain zero coefficients for the given p. (This degeneracy is explicitly verified for the 4 dimensional case in [18].)

The naive way to project out the degenerate modes would be to not sum over the modes of the graviton whose 'angular' modes on H^{D-1} are $r'^{(pu)ij}$ ($p = i(N-1), iN$) in the path integral in the first place. This can be done by taking the $r'^{(pu)ij}$ components with $p = i(N-1), iN$ in the sum (4.18) to be zero. This just gives us,

$$G^{ij}{}_{i'j'}^{\bar{T}}(T,T',l) = \sum_{p:\mathrm{real}} \Phi_p(\bar{T}) Z^{ij}_{(p)i'j'}(l) + \sum_{p=ia_n, i(N-[N-1/2]),\dots i(N-2)} \Phi_p(\bar{T}) Z^{ij}_{(p)i'j'}(l) \tag{4.19}$$

which can be obtained by deforming the initial contour of integration in (4.20) to be C' which is C with two circular contours in the counter-clockwise direction centered at $p = i(N-1)$ and iN added. (We will call these two circles C_N and C_{N-1} respectively.) This is depicted in figure 6. Note that if there are no poles between $i(N-1)$ and iN coming from the reflection coefficient, C' can be taken to be a contour that runs along the real axis of the p plane, with a 'jump' that just passes under $p = i(N-1)$.

Hence the propagator with the redundant modes naively projected out is,

$$G_P^{ij}{}_{i'j'}^{\bar{T}}(T,T',l) = C_0 \int_{C'} dp \mathbb{R} e^{-ip\bar{T}} Y^{ij}_{(p)i'j'}(il) \times (p^2 + (N+1)^2)\Gamma(ip+N-1)\Gamma(-ip+N-1) \tag{4.20}$$

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4.2 The large l limit (again)

Notice that projecting out the given modes do not change the arguments given in section 3.6 that much. Notice that equation (6.39) just gets modified by redefining the contour of integration. That is, I_+ becomes I'_+ where we have the same integrand as I_+ with the different contour, $C'_+ \equiv C_+ + C_N + C_{N-1}$. Also, I_- becomes I'_- which is the integral with the same integrand as I_- but with the different contour of integration, $C'_- \equiv C_- - C_N - C_{N-1}$.

Since only the poles, $p = iN$ and $p = i(N-1)$ cross over from I_- to I_+, we can figure out I'_- and I'_+ easily. First of all,

$$\begin{aligned} I'_{+,1} + I'_{-,1} = & \sum_{n=1}^{\infty} A'_n e^{(-N+n)\bar{T}} G^{ij}_{H\ i'j'}(l, N+n) \\ & + \sum_{n=-\infty}^{0} B'_n e^{(-N+n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-n) \\ & + \sum_{ia_n:\ \text{poles of } \mathbb{R};\ a_n<N} C_n e^{(-N+a_n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-a_n) \\ & + \delta_{N,[N]+1/2}(A'_N G^{ij}_{H\ i'j'}(l, 2N) + A'_{N-1} e^{-\bar{T}} G^{ij}_{H\ i'j'}(l, 2N-1) \\ & + B'_{-(N-1)} e^{-(2N-1)\bar{T}} G^{ij}_{H\ i'j'}(l, 2N-1) + B'_{-N} e^{-2N\bar{T}} G^{ij}_{H\ i'j'}(l, 2N)) \end{aligned} \tag{4.21}$$

We note that $A'_n = A_n$ and $B'_n = B_n$ for $n \neq N, \pm(N-1)$.

For odd dimensions, we get the double pole at $p = iN$ in I'_+;

$$I'_{+,2,odd} = D'_N \bar{T} G^{ij}_{H\ i'j'}(l, 2N) + E'_N \frac{\partial}{\partial \Delta} G^{ij}_{H\ i'j'}(l, \Delta)|_{\Delta=2N} \tag{4.22}$$

For even dimensions, we get double poles at $p = \pm i(N-1)$ and a triple pole at $p = iN$. The double poles give rise to the terms,

$$I_{+,2,even} = D'_{N-1} \bar{T} e^{-\bar{T}} G^{ij}_{H\ i'j'}(l, 2N-1) + E'_{N-1} e^{-\bar{T}} \frac{\partial}{\partial \Delta} G^{ij}_{H\ i'j'}(l, \Delta)|_{\Delta=2N-1} \tag{4.23}$$

$$\begin{aligned} I_{-,2,even} = & J'_{-(N-1)} \bar{T} e^{-(2N-1)\bar{T}} G^{ij}_{H\ i'j'}(l, 2N-1) \\ & + K'_{-(N-1)} e^{-(2N-1)\bar{T}} \frac{\partial}{\partial \Delta} G^{ij}_{H\ i'j'}(l, \Delta)|_{\Delta=2N-1} \end{aligned} \tag{4.24}$$

and the triple pole gives rise to the term,

$$\begin{aligned} I_{-,3,even} = & D'_N \bar{T} G^{ij}_{H\ i'j'}(l, 2N) + F'_N \bar{T}^2 G^{ij}_{H\ i'j'}(l, 2N) \\ & + \left(E'_N \frac{\partial}{\partial \Delta} G^{ij}_{H\ i'j'}(l, \Delta) + G'_N \bar{T} \frac{\partial}{\partial \Delta} G^{ij}_{H\ i'j'}(l, \Delta) \right. \\ & \left. + H'_N \frac{\partial^2}{\partial \Delta^2} G^{ij}_{H\ i'j'}(l, \Delta) \right) |_{\Delta=2N} \end{aligned} \tag{4.25}$$

Note that both in the even and odd dimensional case, we have tamed the logarithmic scaling behavior. Now the asymptotic behavior goes in general like, $\sim e^{-(N-a_n)l}$ where ia_n is the pole of $\mathcal{R}$ with maximum $\Re a_n$, or if there aren't any poles of $\mathcal{R}$ in the upper half plane, it would be like $\sim e^{-Nl}$.

4.3 Treatment of double poles

It seems that we have obtained a well tamed propagator in the previous section, but there is a subtlety we have overlooked. This comes from the fact that we have ignored the contribution of the double pole in (4.18). We have done this by assuming that we can ignore the contribution of gauge dependent modes in the propagator, since these modes can show up by a coordinate transformation.

But it is not clear that this is the thing to do when our propagator is not 'diagonal.' The problem is that the statement that 'we ignore the gauge dependent modes' just restricts the form of "h^{ij}", so schematically, if we denote the modes that should be projected out to be, $|p\rangle$, the gauge condition is just,

$$\langle p|h\rangle = 0 \tag{4.26}$$

for the states $|h\rangle$.

Now if our unprojected propagator is diagonal, i.e. of the form,

$$G = \sum_m M_m |m\rangle\langle m| \tag{4.27}$$

to begin with, the gauge condition can be translated into,

$$G = \sum_{m\neq p} M_m |m\rangle\langle m| \tag{4.28}$$

since

$$G|h\rangle = \sum_m M_m |m\rangle\langle m|h\rangle = \sum_{m\neq p} M_m |m\rangle\langle m|h\rangle \tag{4.29}$$

for $|h\rangle$ satisfying the gauge condition (4.26) anyways.

But if the propagator, as in our case, had the form,

$$G = \sum_{m\neq p} M_m |m\rangle\langle m| + M_0|0\rangle\langle 0| + M_1(|0'\rangle\langle 0| + |0\rangle\langle 0'|) \tag{4.30}$$

where $|0\rangle$ is a mode we projected out, still,

$$G\sum_{m\neq p} a_m|m\rangle = \sum_{m\neq p} M_m a_m|m\rangle + \sum_{m\neq p} M_1 a_m \langle 0'|m\rangle|0\rangle \tag{4.31}$$

so we would have to keep the latter term with M_1, since it has a physical effect on the gauge fixed states.

Suppose the propagator G of the form (4.30) could be written in the particular form,

$$G = \sum_{m\neq p} M_m|m\rangle\langle m| + M_0|0\rangle\langle 0| + M_1 \lim_{\epsilon\to 0}\left(\frac{1}{\epsilon}|0\rangle\langle 0| - \frac{1}{\epsilon}|\epsilon\rangle\langle\epsilon|\right) \tag{4.32}$$

where,

$$\lim_{\epsilon\to 0}|\epsilon\rangle = |0\rangle \tag{4.33}$$

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In this case,

$$|0'\rangle = -\lim_{\epsilon\to 0}\frac{|\epsilon\rangle - |0\rangle}{\epsilon} \tag{4.34}$$

Actually, this is exactly what happens to our propagator as we deform the reflection coefficient of the thin wall. Even if we project out $|0\rangle$, we would still have to keep $|\epsilon\rangle$, but the last term in (4.32) is not well defined, so we regulate it by subtracting $\frac{1}{\epsilon}|0\rangle\langle 0|$ and obtain the M_1 term of (4.30).

So the reason that this piece shows a physical effect becomes clearer in our case. Although the bubble wall bound state mode becomes degenerate with a gauge mode, we cannot treat it as if it did not exist in the first place.

Retaining the double pole contribution, we should write the propagator as,

$$\begin{aligned} G_{P\ \ i'j'}^{ij\ \bar{T}}(T,T',l) = {} & C_0\int_{C'} dp\mathbb{R}e^{-ip\bar{T}}Y_{(p)i'j'}^{ij}(il)(p^2+(N+1)^2)\Gamma(ip+N-1)\Gamma(-ip+N-1)\\ & +K_N\partial_p W_{(p)i'j'}^{ij}(l)|_{p=iN} \end{aligned} \tag{4.35}$$

It is easily verifiable that for large l (and small z in poincare coordinates)

$$\partial_p W_{(p)i'j'}^{ij}(l)|_{p=iN} \sim l t^{ij}_{\ \ i'j'} \sim \ln(|x-x'|/z)t^{ij}_{\ \ i'j'} \tag{4.36}$$

The boundary curvature two point function arising from this piece is non-zero. We will do this calculation in section 5.2.

5 Summary

We have seen in the previous section that we can organize the propagators in the large l limit as a sum of well defined transverse traceless propagators in H^{D-1} coming from single poles in the momentum integral, their normalizable derivatives, and a non-normalizable logarithmic piece coming from the double pole. Put in this form, hopefully, it should be easier to think about what a holographic theory on the S^{D-1} boundary of the "bubble", if exists, would look like.

In this section, we will takes steps to further carry out this effort. We will first summarize the graviton two point function; we will sort out the terms in a well organized way. We will also point out some important features that may have implications about the boundary theory.

5.1 Summary of results

Let's once and for all write down the terms that show up in our 'holographic expansion' of the gauge invariant transverse traceless graviton two point function in a D dimensional CDL instanton background. The full two point function can be written as,

$$G^{ij}_{\ \ i'j'}(T,T',l) = G^{ij\ \delta T}_{\ \ i'j'}(T,T',l) + G_{P\ \ i'j'}^{ij\ \bar{T}}(T,T',l) \tag{5.1}$$

where $G^{ij\ \delta T}_{\ \ i'j'}(T,T',l)$ is the graviton two point function in flat FRW space with no bubble nucleation. We are only intrested in the piece, $G_{P\ \ i'j'}^{ij\ \bar{T}}(T,T',l)$ that arises due to the existence of the CDL instanton.

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This can be written as,

$$G_P^{ij}{}_{i'j'}^{\bar{T}}(T,T',l) = G^{\rm single} + G^{\rm double} \tag{5.2}$$

$G^{\rm single}$ has the same form for both odd and even dimensions. The form for $G^{\rm double}$ differs according to the parity of the dimension.

$G^{\rm single}$ can be written as the following.

$$\begin{aligned}
G^{\rm single} = & \sum_{\substack{n=0 \\ n\neq N,N-1}}^{\infty} A_n e^{-(N+n)\bar{T}} G_H^{ij}{}_{i'j'}(l,N+n) \\
& + \sum_{\substack{n=0 \\ n\neq N,N-1}}^{\infty} B_n e^{-2N\bar{T}} e^{(N+n)\bar{T}} G_H^{ij}{}_{i'j'}(l,N+n) \\
& + \sum_{a_n\neq N} C_n e^{-(N-a_n)\bar{T}} G_H^{ij}{}_{i'j'}(l,N-a_n) \\
& + D_N \partial_p W^{ij}_{(p)i'j'}(l)|_{p=iN}
\end{aligned} \tag{5.3}$$

where ia_n are the poles of $\mathcal{R}(p)$ given by (3.38). We once more recall that we have defined $N \equiv (D-2)/2$.

$G_H^{ij}{}_{i'j'}(l,\Delta)$ are propagators on H^{D-1} defined by (3.63). For $\Delta > N$, these are proportional to massive spin 2 propagators with $m^2 = \Delta(\Delta - 2N)$.

If we write the coordinates of the points in the hyperbolic slices using Poincaré coordinates, (so the coordinates of the two point would be $(T, \vec{x}, z)$ and $(T', \vec{x}', z')$) the propagators showing up in the above sum behave as the following as $z, z' \to 0$;

$$\begin{aligned}
(z^{N+n} z'^{N+n}) & \left[\frac{e^{-(N+n)T} e^{-(N+n)T'}}{(r^2)^{(N+n)}} t^{ij}{}_{i'j'}\right] \\
(z^{N+n} z'^{N+n}) e^{-2NT} e^{-2NT'} & \left[\frac{e^{(N+n)T} e^{(N+n)T'}}{(r^2)^{(N+n)}} t^{ij}{}_{i'j'}\right] \\
(z^{N-a_n} z'^{N-a_n}) & \left[\frac{e^{-(N-a_n)T} e^{-(N-a_n)T'}}{(r^2)^{(N-a_n)}} t^{ij}{}_{i'j'}\right] \\
& \left[\ln(r/z) t^{ij}{}_{i'j'}\right]
\end{aligned} \tag{5.4}$$

where we have defined, $r \equiv |\vec{x} - \vec{x}'|$.

In even dimensions, $G^{\rm double}$ can be written as the following.

$$\begin{aligned}
G^{\rm double} = & E_{N-1} e^{-(2N-1)\bar{T}} [G_H^{ij}{}_{i'j'}(l,2N-1)(a_0' + a_1'\bar{T}) + b_0' \partial_\Delta G_H^{ij}{}_{i'j'}(l,2N-1)] \\
& + E_N e^{-2N\bar{T}} G_H^{ij}{}_{i'j'}(l,2N) \\
& + F_{N-1} e^{-2N\bar{T}} e^{(2N-1)\bar{T}} [G_H^{ij}{}_{i'j'}(l,2N-1)(a_0 + a_1\bar{T}) + b_0 \partial_\Delta G_H^{ij}{}_{i'j'}(l,2N-1)] \\
& + F_N e^{-2N\bar{T}} e^{2N\bar{T}} \Big[G_H^{ij}{}_{i'j'}(l,2N)(c_0 + c_1\bar{T} + c_2\bar{T}^2) \\
& \qquad + \partial_\Delta G_H^{ij}{}_{i'j'}(l,2N)(d_0 + d_1\bar{T}) + e_0 \partial_\Delta^2 G_H^{ij}{}_{i'j'}(l,2N) \Big]
\end{aligned} \tag{5.5}$$

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Where we have defined,

$$\partial^n_\Delta G^{ij}_{H\ \ i'j'}(l,\Delta') \equiv \partial^n_\Delta G^{ij}_{H\ \ i'j'}(l,\Delta)|_{\Delta=\Delta'} \tag{5.6}$$

Taking each term to the boundary we get,

$$\begin{aligned}
(z^{(2N-1)}z'^{(2N-1)}) &\quad \left[\frac{e^{-(2N-1)T}e^{-(2N-1)T'}}{(r^2)^{2N-1}}t^{ij}_{\ \ i'j'}\right] \\
e^{-2NT}e^{-2NT'}\mathcal{O} &\quad \left(\frac{z^{2N+2}z'^{2N+2}}{(r^2)^{2N+2}}\right) \\
(z^{(2N-1)}z'^{(2N-1)})e^{-2NT}e^{-2NT'} &\quad \left[\frac{e^{(2N-1)T}e^{(2N-1)T'}}{(r^2)^{2N-1}}t^{ij}_{\ \ i'j'}\right] \\
(z^{2N}z'^{2N})e^{-2NT}e^{-2NT'} &\quad \left[\frac{e^{2NT}e^{2NT'}}{(r^2)^{2N}}t^{ij}_{\ \ i'j'}\right]\left[\left(a_0+a_1\bar{T}\right)+b_0\ln\left(\frac{r}{z}\right)\right]
\end{aligned} \tag{5.7}$$

In odd dimensions, G^{double} can be written as the following.

$$\begin{aligned}
G^{\text{double}} = & \ E_{N-1}e^{-(2N-1)\bar{T}}G^{ij}_{H\ \ i'j'}(l,2N-1) \\
& +E_N e^{-2N\bar{T}}G^{ij}_{H\ \ i'j'}(l,2N) \\
& +F_{N-1}e^{-2N\bar{T}}e^{(2N-1)\bar{T}}G^{ij}_{H\ \ i'j'}(l,2N-1) \\
& +F_N e^{-2N\bar{T}}e^{2N\bar{T}}[G^{ij}_{H\ \ i'j'}(l,2N)(f_0+f_1\bar{T})+g_0\partial_\Delta G^{ij}_{H\ \ i'j'}(l,\Delta)|_{\Delta=2N}]
\end{aligned} \tag{5.8}$$

Taking each term to the boundary, we obtain,

$$\begin{aligned}
e^{-(2N-1)T}e^{-(2N-1)T'} &\quad \mathcal{O}\left(\frac{z^{2N+1}z'^{2N+1}}{(r^2)^{2N+1}}\right) \\
e^{-2NT}e^{-2NT'} &\quad \mathcal{O}\left(\frac{z^{2N+2}z'^{2N+2}}{(r^2)^{2N+2}}\right) \\
(e^{-2NT}e^{-2NT'})e^{(2N-1)T}e^{(2N-1)T'} &\quad \mathcal{O}\left(\frac{z^{2N+1}z'^{2N+1}}{(r^2)^{2N+1}}\right) \\
(z^{2N}z'^{2N})e^{-2NT}e^{-2NT'} &\quad \left[\frac{e^{2NT}e^{2NT'}}{(r^2)^{2N}}t^{ij}_{\ \ i'j'}\right]
\end{aligned} \tag{5.9}$$

5.2 The logarithmic piece

We first focus on the piece,

$$\partial_p W^{ij}_{(p)i'j'}(l)|_{p=iN} \sim l t^{ij}_{\ \ i'j'} \tag{5.10}$$

The natural thing to do with this is to calculate the ‘curvature two point function’ coming from this piece. To explain a bit more, if we assume that this piece corresponds to a two point function $< h^{ij}h_{ij} >$ of some transverse traceless operator on the boundary, we would like to see what the gauge invariant two point function, $\nabla_i\nabla_j\nabla^{i'}\nabla^{j'} < h^{ij}h_{i'j'} >$ is.

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Let's first try to find the relevant components of this by explicitly writing this in Poincaré coordinates on the hyperboloid. We do so because it is convenient to see the behavior at the boundary in these coordinates. To write it once more, the Poincaré coordinate on the hyperboloid is,

$$ds^2 = \frac{1}{z^2}(dz^2 + dx_1^2 + \cdots + dx_{D-2}^2) \tag{5.11}$$

The boundary is at $z = 0$. Let's consider two points $(x_1, \dots, x_{D-2}, z)$, $(-x_1, \dots, -x_{D-2}, z)$ and look at the $z \to 0$ limit. In the Poincaré coordinates, the geodesic that connects two points on the boundary is a half circle. So considering the given two points, the unit tangent vector n_i and n'_i at $(x_1, \dots, x_{D-2}, z)$ and $(-x_1, \dots, -x_{D-2}, z)$ respectively is,

$$\begin{pmatrix} n_z \\ n_{x_i} \end{pmatrix} = \begin{pmatrix} -r'/zr \\ x_i/rr' \end{pmatrix}, \quad \begin{pmatrix} n_{z'} \\ n_{x'_i} \end{pmatrix} = \begin{pmatrix} -r'/zr \\ -x_i/rr' \end{pmatrix} \tag{5.12}$$

where we define,

$$r' = \sqrt{x_1^2 + x_2^2 + \cdots x_{D-2}^2}, \qquad r = \sqrt{z^2 + s'^2} \tag{5.13}$$

for convenience.

The parallel transport operator in our case is just a rotation matrix, which is,

$$\begin{pmatrix} g_z{}^{z'} & g_z{}^{x'_j} \\ g_{x_i}{}^{z'} & g_{x_i}{}^{x'_j} \end{pmatrix} = \frac{1}{r^2}\begin{pmatrix} -r^2+2z^2 & -2zx_1 & \cdots & -2zx_{D-2} \\ 2zx_1 & r^2-2x_1^2 & -\cdots & -2x_1x_{D-2} \\ \vdots & \vdots & \ddots & \vdots \\ 2zx_{D-2} & -2x_{D-2}x_1 & \cdots & r^2-2x_{D-2}^2 \end{pmatrix} \tag{5.14}$$

Also, the geodesic distance between the two points may be calculated as,

$$l = 2\ln\frac{r+r'}{z} \tag{5.15}$$

Calculating $t^{ij}{}_{i'j'}$ from this, we find that if the correlator has at least one z index, it is of order $O(z)$. Hence, in the limit $z \to 0$, we find that the only surviving components of $t^{ij}{}_{i'j'}$ are those with all the indicies are in the $D-2$ plane on the boundary, that is,

$$t^{ij}{}_{i'j'} \sim \mathcal{O}(z) \qquad \text{for } z \to 0 \ \text{ unless } i, j, i', j' \neq z \tag{5.16}$$

Hence when we take the correlator to the boundary, the only surviving tensors components (5.10) are those whose indices are all along the boundary directions. Also, in this limit, $l = 2\ln(2r'/z)$

We can actually write the form of $t^{ij}{}_{i'j'}$ on the boundary plane from direct calculation which yields,

$$\begin{aligned} &t^{ij}{}_{i'j'}((x_1, \dots, x_{D-2}), (-x_1, \dots, -x_{D-2})) \\ &\qquad = \delta_{ij}\delta_{i'j'} \ -N\left(\delta_{ii'} - \frac{2x_ix_{i'}}{r'^2}\right)\left(\delta_{jj'} - \frac{2x_jx_{j'}}{r'^2}\right) \\ &\qquad\qquad -N\left(\delta_{ij'} - \frac{2x_ix_{j'}}{r'^2}\right)\left(\delta_{ji'} - \frac{2x_jx_{i'}}{r'^2}\right) + O(z) \end{aligned} \tag{5.17}$$

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when i, j, i', j' are all along the direction of the boundary. Using translational invariance in the boundary space we obtain,

$$
\begin{aligned}
&t^{ij}{}_{i'j'}((x_1,\dots,x_{D-2}),(y_1,\dots,y_{D-2})) \\
&\quad = \delta_{ij}\delta_{i'j'} - N\left(\delta_{ii'} - \frac{2(x_i-y_i)(x_{i'}-y_{i'})}{R'^2}\right)\left(\delta_{jj'} - \frac{2(x_j-y_j)(x_{j'}-y_{j'})}{R'^2}\right) \\
&\qquad - N\left(\delta_{ij'} - \frac{2(x_i-y_i)(x_{j'}-y_{j'})}{R'^2}\right)\left(\delta_{ji'} - \frac{2(x_j-y_j)(x_{i'}-y_{i'})}{R'^2}\right)
\end{aligned}
\tag{5.18}
$$

by replacing $2x_i$ by $x_i - y_i$.

Note that we have newly defined,

$$R'^2 = (x_1-y_1)^2 + \cdots + (x_{D-2}-y_{D-2})^2 \tag{5.19}$$

which satisfies $l = 2\ln(R'/z)$.

Let's attempt to calculate a gauge invariant quantity, the $D-2$ dimensional scalar curvature of the graviton fluctuation. Since a traceless perturbation h_{ij} of the curvature in a flat background yields,

$$C \propto \partial_i \partial_j h^{ij} \tag{5.20}$$

we get,

$$< C(x)C(y) >= \partial_i\partial_j\partial^{i'}\partial^{j'} c_0 l t^{ij}{}_{i'j'} = \frac{c_1}{R'^4} + \frac{c_2\ln(R'/z)}{R'^4} \tag{5.21}$$

where,

$$c_1 = -16(2N-1)(3N-2)(4N^2-7N+1)c_0 \tag{5.22}$$

$$c_2 = -64N(N-1)(N-2)(2N-1)^2 c_0 \tag{5.23}$$

It's worth noting that the $\ln(R'/z)/R'^4$ term vanishes only for $D = 3, 4, 6$, and that for $D = 3$, the curvature vanishes altogether. (We've ignored the $D = 2$ case since this calculation doesn't make sense if coordinates are not defined at all in the first place.)

5.3 Existence of a stress-energy tensor

We notice from the expression given in section 5.1 (namely equations (5.5) and (5.8)) that we have a dimension $2N = (D-2)$ transverse traceless tensor propagator in piece in H^{D-1}. That is, we have the pieces which in Poincaré coordinates, ignoring the T dependence, behaves like

$$G_H^{ij}{}_{i'j'}(l, 2N) \sim z^{2N} z'^{2N}(x-x')^{-4N} t^{ij}{}_{i'j'} \tag{5.24}$$

as $z, z' \to 0$.

Writing this for two points at equal z we get,

$$G_H^{ij}{}_{i'j'}(l, 2N) \sim \frac{z^{4N}}{R'^{4N}} t^{ij}{}_{i'j'} \tag{5.25}$$

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By direct calculation, it is verified that this piece is transverse-traceless on the $(D-2)$ dimensional boundary, namely that,

$$\frac{1}{R'^{4N}} t^{i}{}_{ii'j'} = 0 \tag{5.26}$$

$$\partial_i \left(\frac{1}{R'^{4N}} t^{ij}{}_{i'j'} \right) = 0 \tag{5.27}$$

Actually we see that this conincides with the expression for the two point function of the stress energy tensor of a CFT (namely equation (2.18)) given in [19].

We do not want to hastily imply that the piece that shows up in our expansion is a stress engergy tensor on the boundary, but if we assume some kind of holographic correspondence there seems to exist a dimension $(D-2)$ transverse traceless tensor on a $(D-2)$ dimensional boundary theory.

Note that from looking at equation (5.7) there seems to be some kind of obstruction of this term that comes from the pole $p = iN$. We will try to address this issue in the final section.

5.4 Odd and even dimensions

Since we know that gravity behaves very differently in even and odd dimensions, we would expect the behavior of the propagator to be drastically different for the two cases, which indeed it is. The most dramatic difference would be that the number of poles of the reflection coefficient for even dimensions is finite (as the reflection coefficient becomes a rational function with respect to p) whereas in odd dimensions it is infinite. Hence if we want to think about some holographic correspondence, an infinite number of operators with different dimensions seems to come at play for odd dimensions whereas for even dimensions number seems finite.

Also in odd dimensions some values of X_0 seem to give rise to an infinite number of complex poles for $\mathcal{R}$. This happens at a sharp point, namely at $X_0 = 0$. If there is indeed some kind of holographic dual theory that lives at the boundary S^{D-2} that is dual to the CDL gravity theory, this suggests that there might be some phase transition or duality in that theory for odd dimensions, whereas for even dimensions, where all the poles stay on the imaginary axis for all values of X_0, nothing of the sort seems to happen.

6 The scalar propagator

We will follow the exact steps taken as we have with the graviton propagator in obtaining the propagator for an arbitrary minimally coupled scalar ψ in the given background.

6.1 The equation of motion

We first consider when ψ is massless. We first define

$$\chi = a^N(X)\psi \tag{6.1}$$

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Reusing the notations we have used for the graviton, the relevant part of the action turns out to be,

$$S = \frac{1}{2}\int dX d\Omega_{D-1}\sqrt{\tilde{g}}\chi\left[-\partial_X^2 + \mathrm{U}(X) - \tilde{\Box}\right]\chi \tag{6.2}$$

Hence by defining,

$$\hat{G}(X_1, X_2, \Omega_1, \Omega_2) = a^N(X_1)a^N(X_2) < \chi(X_1, \Omega_1)\chi(X_2, \Omega_2) > \tag{6.3}$$

we get,

$$\left[-\partial_{X_1}^2 + \mathrm{U}(X_1) - \tilde{\Box}_1\right]\hat{G}(X_1, X_2, \Omega_1, \Omega_2) = \frac{1}{\sqrt{\tilde{g}}}\delta(X_1 - X_2)\delta(\Omega_1, \Omega_2) \tag{6.4}$$

6.2 Decomposition

Due to the $O(D-1)$ symmetry, the Green's function G can only be a function of X, X' and the geodesic distance $\mu(\Omega_1, \Omega_2)$ between the two points on the $(D-1)$ sphere. Hence, we may write the solution for the equation (6.4) simply as,

$$\hat{G}(X, X', \mu) = \sum_{p=iN}^{+i\infty} G_p^s(X, X')W_{(p)}(\mu) \tag{6.5}$$

for $G_p^s(X, X')$ and $W_{(p)}(\mu)$ which we will define below.

We define G_p^s to satisfy equation (3.19). The reason we didn't just put G_p^s equal to G_p defined in (3.39) is because G_p obtained as (3.41) for $p = iN$ is singular due to the pole of $\mathcal{R}$ at $p = iN$. G_p has a simple pole at $p = iN$ and the residue $R(X, X')$ of this pole satisfies the equation,

$$[-\partial_X^2 + \mathrm{U}(X)]R(X, X') = 0 \tag{6.6}$$

This is because $R(X, X')$ is normal at $X = X'$. (For example, when $X, X' < X_0$ it is an exponential of $X + X'$ so it behaves normally.) Hence we may define

$$G_{iN}^s(X, X') \equiv \lim_{p\to iN}\left(G_p(X, X') - \frac{\mathrm{Res}_{p=iN}G_p(X, X')}{p - iN}\right) \tag{6.7}$$

and G_{iN}^s will still satisfy equation (3.19) for $p = iN$. For $p \neq iN$, we may set G_p^s safely equal to G_p. Hence for $X, X' < X_0$ we get,

$$G_p^s(X, X') \equiv \frac{i}{2p}(e^{ip\delta X} + \mathbb{R}(p)e^{-ip\bar{X}}) \tag{6.8}$$

$$G_{iN}^s(X, X') \equiv \frac{i}{2p}\left(e^{ip\delta X} + \left(\mathbb{R}(p) - \frac{\mathrm{Res}_{p=iN}\mathbb{R}(p)}{p - iN}\right)e^{-ip\bar{X}}\right) \tag{6.9}$$

$W_{(p)}(\mu)$ is a scalar function only dependent upon $\mu(\Omega_1, \Omega_2)$. $W_{(p)}(\mu)$ is defined by,

$$W_{(p)}(\mu) = \sum_u q^{(pu)}(\Omega)q^{(pu)}(\Omega')^* \tag{6.10}$$

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where $q^{(pu)}$ are transeverse traceless eigenmodes of

$$\widetilde{\Box} q^{(pu)} = (N^2 + p^2) q^{(pu)} \tag{6.11}$$

which are normalized so that

$$\int d^{D-1}x \sqrt{\tilde{g}} q^{(pu)} q^{(p'u')*} = \delta^{pp'} \delta^{uu'} \tag{6.12}$$

Note that we denoted all the quantum numbers other than p needed to specify the mode q as u. $W_{(p)}(\mu)$ satisfies,

$$\widetilde{\Box} W_{(p)}(\mu) = (N^2 + p^2) W_{(p)}(\mu) \tag{6.13}$$

On S^{D-1}, we get eigenmodes for the p values, $p = iN, i(N+1), \ldots$, so by completeness of the basis,

$$\sum_{p=iN}^{+i\infty} W_{(p)}(\mu(\Omega, \Omega')) = \delta(\Omega, \Omega')/\sqrt{\tilde{g}} \tag{6.14}$$

From equations (6.13), (6.14), and (3.19), we see that indeed (6.5) solves (6.4).

6.3 $W_{(p)}(\mu)$

The equation for $W_{(p)}(\mu)$ can be written out as in [14], which is,

$$W''_{(p)}(\mu) + (D-2) \cot \mu G'(\mu) - (N^2 + p^2) G(\mu) = 0 \tag{6.15}$$

This can be solved to be,

$$W_{(p)}(\mu) = K_p F\left(N + ip, N - ip; N + \frac{1}{2}; 1 - z\right) \quad \text{for} \quad z = \cos^2 \frac{\mu}{2} \tag{6.16}$$

where from (6.10) we see that W_p to be non-singular at $\mu = 0$. K_p can be calculated from (6.16) and (6.10)

$$\begin{aligned} K_p \frac{2\pi^{D/2}}{\Gamma(D/2)} &= \int d^{D-1}\Omega \sqrt{\tilde{\gamma}} W_{(p)}(\Omega, \Omega) \\ &= -\frac{2ip(p^2 + (N-1)^2)\Gamma(-ip + N - 1)}{(D-2)!\Gamma(-ip - N + 2)} \end{aligned} \tag{6.17}$$

by the degeneracy of the p mode [15]. Hence we obtain,

$$\begin{aligned} W_{(p)}(\mu) = & \left[-\frac{i\Gamma(D/2)}{\pi^{D/2}}\right] \frac{p(p^2 + (N-1)^2)\Gamma(-ip + N - 1)}{(D-2)!\Gamma(-ip - N + 2)} \\ & \times F\left(N + ip, N - ip; N + \frac{1}{2}; 1 - z\right) \end{aligned} \tag{6.18}$$

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6.4 Massive scalar propagators in $H^{(D-1)}$

The equation for the propagator for a massive scalar in H^{D-1} with curvature radius $R^2 = -1$ is,

$$\left(-\widetilde{\Box}_1 - m^2\right) G_H(l(\Omega_1, \Omega_2), m^2) = \frac{1}{\sqrt{\tilde{\gamma}}} \delta(\Omega_1, \Omega_2) \tag{6.19}$$

This is solved in [14] to be,

$$G_M(l, m^2) = \left[\frac{\Gamma(N - ip)\Gamma(1/2 - ip)}{\Gamma(1 - 2ip)\pi^{(D-1)/2} 2^{(D-1)}}\right] \left(\frac{1}{z}\right)^{N-ip} F\left(N - ip, 1/2 - ip; 1 - 2ip; \frac{1}{z}\right) \tag{6.20}$$

where $p = i\sqrt{N^2 + m^2}$. For $z \to \infty$

$$G_M(l, m^2) \sim \left(\frac{1}{z}\right)^{N-ip} \tag{6.21}$$

As in the case for the graviton, we define,

$$G_H(l, \Delta) = \left(\frac{1}{z}\right)^{\Delta} F\left(\Delta, -N + \frac{1}{2} + \Delta; -2N + 1 + 2\Delta; \frac{1}{z}\right) \tag{6.22}$$

We note that,

$$G_H(l, \Delta) \sim e^{-\Delta l} \tag{6.23}$$

for non-problematic Δ and that

$$G_H(l, \Delta) \propto G_M(l, \Delta(\Delta - 2N)) \tag{6.24}$$

for $\Delta > N$.

As in the graviton case, $G_H(l, \Delta)$ is singular when $\Delta = N - n$ for positive integer n. Following the exact same steps taken in appendix E we see that,

$$G_H(l, N - ip) = \frac{K_{s,n}}{p + in} G_H(l, N + n) + H_0(l, N - n) + H_1(l, N - n) + \mathcal{O}((p + in)^2) \tag{6.25}$$

where H_0 and H_1 at large l behave as,

$$H_0(l, N - n) \sim e^{-(N-n)l} \tag{6.26}$$

$$H_1(l, N - n) \sim l e^{-(N-n)l} \tag{6.27}$$

6.5 Analytic continuation

The sum (6.5) may be expressed as,

$$\hat{G}(X, X', \mu) = \int_{C_{s1}} \frac{dp}{2\pi i} \frac{\Gamma(-ip - N + 1)\Gamma(ip + N)}{(-1)^{-ip-N}} \times G_p^s(X, X') W_{(p)}(\mu) \tag{6.28}$$

where the contour C_{s1} is defined to be one that comes down from $i\infty$ on the left side of the imaginary axis of the complex p plane, and pivots around $p = iN$ to go back to $i\infty$ by the right side of the imaginary axis.

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Plugging in (3.41) in to this equation we obtain,

$$\hat{G}(X,X',\mu) = \int_{C_{s1}} \frac{dp}{4\pi p} \frac{\Gamma(-ip-N+1)\Gamma(ip+N)}{(-1)^{-ip-N}} \\ \times(e^{ip\delta X} + \mathbb{R}(p)e^{-ip\bar{X}})W_{(p)}(\mu) \\ +Ae^{N\bar{X}} + Be^{N\bar{X}}\bar{X} + Ce^{N\bar{X}}\frac{\partial}{\partial p}W_p(\mu)|_{p=iN} \tag{6.29}$$

where the additional term comes from the double pole arising from the additional term in G_{iN} given in (6.9). Note that $W_{iN}(\mu)$ is constant.

We focus our attention on the integral of latter term(+the residual terms), where the first term just gives the scalar propagator in flat space. By essentially the same arguments given in the spin 2 case, the contour of integration for the latter terms can be safely deformed to the contour C_s, which we define to run along the real axis of the p plane, with a 'jump' just under $p = iN$. We get,

$$\hat{G}^{\bar{X}}(X,X',\mu) = \int_{C_s} \frac{dp}{4\pi p} \frac{\Gamma(-ip-N+1)\Gamma(ip+N)}{(-1)^{-ip-N}} \\ \times\mathbb{R}(p)e^{-ip\bar{X}}W_{(p)}(\mu) \\ +Ae^{N\bar{X}} + Be^{N\bar{X}}\bar{X} + Ce^{N\bar{X}}\frac{\partial}{\partial p}W_p(\mu)|_{p=iN} \tag{6.30}$$

After the analytic continuation,

$$X = T + i\frac{\pi}{2}, \qquad \mu = il \tag{6.31}$$

we finally obtain,

$$G^{\bar{T}}(T,T',l) = C_{s0}\int_{C_s} dp\Gamma(ip+N)\Gamma(-ip+N)\mathbb{R}e^{-(N+ip)\bar{T}}Y_{(p)}(il) \\ +A' + B'\bar{T} + C'\frac{\partial}{\partial p}Y_p(il)|_{p=iN} \tag{6.32}$$

where we have conveniently defined,

$$Y_{(p)}(il) \equiv F\left(N+ip, N-ip; N+\frac{1}{2}; 1-z\right)\Big|_{z=\cosh^2\frac{l}{2}} \tag{6.33}$$

and we have gotten rid of the hat on the propagator by multiplying $e^{-N\bar{T}}$.

6.6 A gauge argument

Let's examine the terms,

$$A' + B'\bar{T} + C'\frac{\partial}{\partial p}Y_p(il)|_{p=iN} \tag{6.34}$$

of (6.32).

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The first two terms, $A' + B'T_1 + B'T_2$ vanish when we take derivatives with respect to both points showing up in the two point function. In other words, these terms are pure gauge. Getting rid of this term we can write,

$$G^{\bar{T}}(T,T',l) = C_{s0} \int_{C_s} dp \Gamma(ip+N)\Gamma(-ip+N)\mathbb{R}e^{-(N+ip)\bar{T}} Y_{(p)}(il) + K_{sN} \frac{\partial}{\partial p} Y_{(p)}(il)|_{p=iN} \tag{6.35}$$

Note that

$$I_{sN} \equiv K_{sN} \frac{\partial}{\partial p} Y_{(p)}(il)|_{p=iN} \sim l \tag{6.36}$$

for large l.

6.7 The large l limit

Due to the identity between hypergeometric functions,

$$Y_p(il) = \frac{\Gamma(N+\frac{1}{2})\Gamma(-2ip)}{\Gamma(N-ip)\Gamma(\frac{1}{2}-ip)} G_H(l, N+ip) + \frac{\Gamma(N+\frac{1}{2})\Gamma(2ip)}{\Gamma(N+ip)\Gamma(\frac{1}{2}+ip)} G_H(l, N-ip) \tag{6.37}$$

hence the first term in (6.35) can be written as,

$$\begin{aligned} \hat{G}^{\bar{T}} &= C_{s0} \int_{C_s} dp \mathbb{R}e^{-(N+ip)\bar{T}} \left[\frac{\Gamma(-ip)\Gamma(ip+N)}{2^{-2ip-1/2}} G_H(l, N+ip) \right. \\ &\qquad \left. + \frac{\Gamma(ip)\Gamma(-ip+N)}{2^{2ip-1/2}} G_H(l, N-ip) \right] \end{aligned} \tag{6.38}$$

Define the contour C_{s-} to be the contour coming from $-i\infty$ on the left side of the imaginary axis, pivoting just under $p = iN$ and going back down to $-i\infty$ on the right side of the imaginary axis. Define the contour C_{s+} to be the contour coming from $i\infty$ on the left side of the imaginary axis, pivoting around $p = iN$ and going back up to $i\infty$ on the right side of the imaginary axis. Then we may deform the contour of integration for each term to be,

$$\begin{aligned} G^{\bar{T}} &= C_{s0} \int_{C_{s-}} dp \mathbb{R}e^{(-N-ip)\bar{T}} \frac{\Gamma(-ip)\Gamma(ip+N)}{2^{-2ip-1/2}} G_H(l, N+ip) \\ &\quad + C_{s0} \int_{C_{s+}} dp \mathbb{R}e^{(-N-ip)\bar{T}} \frac{\Gamma(ip)\Gamma(-ip+N)}{2^{-2ip-1/2}} G_H(l, N-ip) \\ &\equiv I_{s-} + I_{s+} \end{aligned} \tag{6.39}$$

The poles of the integrand of I_{s+} are given as the following.

1. $p = in$ for integers n.
2. $p = -i(N+n)$ for non-negative integer n.
3. The poles of $\mathcal{R}$.

The only feature we should pay attention to is that $p = iN$ is a double pole for even dimensions. All other poles that contriubte are all simple poles.

The poles of the integrand of I_{s-} are given as the following.

1. $p = in$ for integers n.

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2. $p = i(N+n)$ for non-negative integer n.

3. The poles of $\mathcal{R}$.

The poles that contribute will in general all be simple poles.

We can finally write out,

$$\begin{aligned} I_{s-} + I_{s+} + I_{sN} = & \sum_{n=1}^{\infty} A_{sn} e^{(-N+n)\bar{T}} G_H(l, N+n) \\ & + \sum_{n=-\infty}^{0} B_{sn} e^{(-N+n)\bar{T}} G_H(l, N-n) \\ & + \sum_{ia_n:\text{ all poles of }\mathbb{R}} C_{sn} e^{(-N+a_n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-a_n) \\ & + \delta_{N,[N]}(D_{sN}\bar{T} G_H(l,2N) + E_{sN} \frac{\partial}{\partial \Delta} G_H(l,\Delta)|_{\Delta=2N}) \\ & + K_{sN} \frac{\partial}{\partial p} Y_{(p)}(il)|_{p=iN} \end{aligned} \tag{6.40}$$

Note that at large l,

$$\frac{\partial}{\partial \Delta} G_H(l,\Delta)|_{\Delta=2N} \sim l e^{-2Nl} \tag{6.41}$$

6.8 The accidental double pole

One thing we must mention about the expression (6.40) for the scalar propagator is that the double pole $p = iN$ that arises is purely a coincidence coming from our assumption that the scalar is massless on both sides of the bubble wall. There is no reason that this should be the case in general for minimally coupled scalars.

One minimally coupled scalar we know that exists in our model is the scalar field ϕ, namely the modulus field. In the case of this field, it is certainly natural to assume a mass at least in the false vaccum. This would modify (6.4) so that $\mathrm{U}(X) \to \mathrm{U}(X) + m^2 a(X)^2 \Theta(X - X_0)$. $G_p(X, X')$ used in the sum (6.4) would have to be modified. If we assume the scalar to be massless in the true vacuum, it would still be of the form (3.41) but the reflection coefficient, $\mathbb{R}(p)$ would be modified. In fact, as pointed out in [5], this shifts the pole at $p = iN$ to $p = i(N - \epsilon)$ where $\epsilon > 0$.

Hence in general, the expression (6.40) would be modified to

$$\begin{aligned} G^{\bar{T}} = & \sum_{n=1}^{\infty} A_{sn} e^{(-N+n)\bar{T}} G_H(l, N+n) \\ & + \sum_{n=-\infty}^{0} B_{sn} e^{(-N+n)\bar{T}} G_H(l, N-n) \\ & + \sum_{ia'_n:\text{ all poles of }\mathbb{R}'} C_{sn} e^{(-N+a'_n)\bar{T}} G^{ij}_{H\ i'j'}(l, N-a'_n) \end{aligned} \tag{6.42}$$

Note that for the graviton case, nothing of this sort happens; the graviton is massless on both sides of the wall. The reflection coefficient $\mathbb{R}(p)$ is given exactly by (3.37)

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rendering the pole at $p = iN$ to be at least doubly degenerate. Unlike for the case of the scalar that provides the tunneling, the logarithmic piece seems to be a crucial element of the graviton propagator.

7 Speculation and outlook

7.1 Holographic correspondence

For the moment, let's be optimistic and assume that an AdS/CFT like correspondence exists for a bulk theory in the flat time-like region of the D dimensional CDL background and the S^{D-2} boundary at spacelike infinity. In this section, we will try to make some suggestions of what such a theory would look like.

For the sake of simplicity of argument, let's assume the scalar mass is zero on both sides of the wall. This is because we don't want to introduce a mass scale other than the size of the wall, which comes from the geometry of the background.

FSSY suggested in [5] that in the 4D case the field theory in the time-like flat bulk corresponds to a Liouville theory on the S^2 boundary. In the process they have identified the time coordinate with the Liouville field of the boundary($L = 2T$). In that sense, we can view time being emergent from a Liouville field.

We can certainly see something similar in general dimensions. By writing out the two point functions as we have, (more precisely, by arranging the terms according to the scaling behavior with respect to $e^{\bar{T}}$) we see that the two point functions(both for the spin 2 and 0 case) can be basically written as a sum of three kinds of terms,

$$\begin{aligned}
&e^{-(N+n)T_1}e^{-(N+n)T_2}G_H(l, N+n) && n : \text{non-negative integers}\\
&e^{-(N-a_n)T_1}e^{-(N-a_n)T_2}G_H(l, N-a_n) && ia_n : \text{poles of } \mathcal{R}\\
&e^{-2N\bar{T}}e^{(N+n)T_1}e^{(N+n)T_2}G_H(l, N+n) && n : \text{non-negative integers}
\end{aligned} \tag{7.1}$$

where $G_H(l, \Delta)$ is a dimension Δ propagator with a given spin on H^{D-1}. (There are terms that certainly don't fit in to this framework, and we will discuss them later.) If we assume the existence of a holographic duality of a field theory in this background, it is very tempting to view the time T as a dilatonic field on Σ by writing the propagator out this way.

Indeed, if we take a slice of our space, $(T(x), x)$ where $x = (\vec{x}, z)$ are the Poincare coordinates on H^{D-1}, the propagator restricted to this slice can be written as a sum of

$$e^{-(N+n)T(x_1)}e^{-(N+n)T(x_2)}G_H(x_1, x_2, N+n) \tag{7.2}$$

$$e^{-(N-a_n)T(x_1)}e^{-(N-a_n)T(x_2)}G_H(x_1, x_2, N-a_n) \tag{7.3}$$

$$e^{-2N\bar{T}}e^{(N+n)T(x_1)}e^{(N+n)T(x_2)}G_H(x_1, x_2, N+n) \tag{7.4}$$

If we take these to Σ by taking $z \to 0$ and stripping away the z dependence by defining

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$T(\vec{x}) \equiv \lim_{z\to 0} T(\vec{x}, z)$ we get,

$$\frac{e^{-(N+n)T(\vec{x_1})}e^{-(N+n)T(\vec{x_2})}}{|\vec{x_1}-\vec{x_2}|^{2(N+n)}}(t^{ij}{}_{i'j'}) \tag{7.5}$$

$$\frac{e^{-(N-a_n)T(\vec{x_1})}e^{-(N-a_n)T(\vec{x_2})}}{|\vec{x_1}-\vec{x_2}|^{2(N-a_n)}}(t^{ij}{}_{i'j'}) \tag{7.6}$$

$$(e^{-2N\bar{T}})\frac{e^{(N+n)T(\vec{x_1})}e^{(N+n)T(\vec{x_2})}}{|\vec{x_1}-\vec{x_2}|^{2(N+n)}}(t^{ij}{}_{i'j'}) \tag{7.7}$$

where $t^{ij}{}_{i'j'}$ given by (5.18) is multiplied to each scalar part for the tensor two point function. By (5.16), only the components with indices in the tangential directions survive at the boundary. $t^{ij}{}_{i'j'}$ actually is proportional to that given in equation (2.18) of [19]. We can see that the first two terms are of the same form as two point functions of (quasi-)primary operators of a CFT given in [19] in a dilatonic background $2T(\vec{x})$, and the last with $-2T(\vec{x})$ multiplied by an additional prefactor.

What these two kinds of propagators mean is not clear, but it is possible that the graviton and scalar field correspond to a sum of spin 2 and spin 0 operators living on the boundary with definite scaling dimensions.

One imaginable scenario is that we have 2 CFTs, CFT_1 and CFT_2 coupled to possibly a gravity theory such that the action is given by,

$$\int \mathcal{L}_1\left(\Omega_1 = e^{2T}\right) + \int \mathcal{L}_2\left(\Omega_2 = e^{-2T}\right) \tag{7.8}$$

Where $\mathcal{L}_i(\Omega_i)$ denotes the CFT_i lagrangian with local scaling Ω_i. This is due to the fact that we have two distinguishable contributions to our propagator: the waves going toward the boundary wall and the waves coming from the boundary wall. If our bulk field corresponds to an operator sum,

$$\phi \to \mathcal{O} \equiv \sum_{\Delta_1} \mathcal{O}_1(\Delta_1) + \sum_{\Delta_2} e^{-2NT}\mathcal{O}_2(\Delta_2) \tag{7.9}$$

with $\mathcal{O}_1$ being primary operators in CFT_1 and $\mathcal{O}_2$ being primary operators in CFT_2, the two point function of $\mathcal{O}$, with fixed $T(\vec{x})$ would indeed look like something we have.[4]

A few comments are to be made. Trying to interpret the two point function this way, we notice that we have operators that aren't of dimension $N+n$, namely ones with dimension $N-a_n$ where a_n depends on the bubble wall position. (More precisely put, a_n are real poles of the function,

$$\frac{F(-N+1, N+1; 1+a_n; t)}{F(-N, N; 1+a_n; t)} \tag{7.10}$$

for $t = \frac{e^{-X_0}}{2\cosh X_0}$.) This means that we have operators with anomalous dimensions, depending on a tunable parameter of the theory, X_0. If we give a mass to the scalar, the terms

[4] An alternative interpretation is offered in [6] where it is conjectured that there is only one CFT and $\mathcal{O}_1$ and $\mathcal{O}_2$ are interpreted as renormalization invariant("proactive") and renormalization covariant("reactive") operators.

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showing up in the scalar propagator would depend on the mass as well. But the point is that we have a dimensionful parameter coming from the geometry of the background, and that the anomolous dimensions of operators are related to this by an analytic function.

Also, thinking of graviton fields on the boundary as dimension 0 operators, we have a natural interpretation for the logarithmic term. As we can see from the terms showing up in the expansion for the graviton propagator written out in section 5.1, it can be written out as a sum of propagators that are well behaved at the boundary, plus a logarithmic(dimension zero) piece. We've seen in section 5.2 that this piece has a fluctuation the size of the background curvature. This suggests that the boundary theory should have geometric fluctuations, which indeed is coherent with the conjecture that T is emergent from a dilatonic field on the boundary theory. Actually, to stretch our conjecture a bit more, it is possible that CFT_1 mentioned above contains gravity where the fluctuation of T corresponds to the dilaton. All such speculation is coherent with the two point function we have obtained, but much more evidence would be needed to back up this proposal.

We also note that the coefficients showing up for the three kinds of propagators in the propagator sum depend on the reflection coefficient, and in the thin wall limit, ultimately on the bubble wall position. If we assume that indeed our bulk fields correspond to a sum of operators on the boundary, then how they are summed to give a corresponding bulk field is dependent upon the bubble wall position.

Another issue we must address are the irregular correlators that show up for operators of dimension, $\Delta = (D-2), (D-3)$. These can be seen in equations, (5.7) and (5.9). The propagator corresponding to $\Delta = (D-3)$ is easy to think about. In the even dimensional case, they just are propagators of operators of dimension $(D-3)$. In the odd dimensional case, the leading order behavior is of dimension $(D-1)$ ($\sim z^{D-1}$) which doesn't match its scaling dimension with respect to T. We don't quite understand this piece and will ignore it, as it disappears faster than it should as $z \to 0$. Under this prescription, $N + (N-1) = (D-3)$ is not a special case. In even dimensions, $(N-1)$ is an integer, so it is natural for an $N + (N-1)$ dimensional operator to show up in the sum. In odd dimensions, $(N-1)$ is not an integer, so an $N + (N-1)$ dimensional operator doesn't show up in the sum.

The interpretation of the dimension $(D-2)$ piece seems to be trickier. Just as with the $(D-3)$ dimensional propagators, let's choose to discard the pieces with leading order behavior $\sim z^D$. Then, if we try to interpret it as a stress energy tensor as we have suggested in section 5.3, we see that it only exists for CFT_2, and in the even dimensional case, is obstructed by a logarithmic term. The lograrithmic term causes a problem because it renders the stress energy tensor to be non-transverse. How to treat this is not entirely clear at the moment. This is because we have a dimension zero operator in CFT_1 with the same $e^{\bar{T}}$ power as the stress energy tensor of CFT_2. It would be comforting if we could just get rid of the logarithmic term by claiming that it comes from the dimension zero operator and ignore it, but at the moment it stands as a term we have to deal with.

Also, the fact that a dimension $(D-2)$ operator doesn't show up for CFT_1 is interesting. We have conjectured that gravity would live in CFT_1, so it might be that only CFT_1 respects the full diffeomorphism invariance rendering $T_1^{ij} = 0$, and CFT_2 only responds to dilatonic fluctuations.

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There is a different interpretation of this from the framework of [6]. In this case, there is only one stress energy tensor for the theory in the first place. The existence of a non-zero stress energy tensor will be an indication that the Liouville field has decoupled from the rest of the theory at some fixed point.

As we have already mentioned, the boundary theory has a tunable parameter: the bubble wall position. We have seen that the bubble wall position determines the correspondence between fields and operator sums. It also determines the dimensions of operators that come from the pole of the reflection coefficient. We have seen in section 5.4 that this is conspicuous in odd dimensions, as the reflection coefficient has an infinite number of poles in this case. Tuning the bubble wall position also seems to trigger some kind of phase transition in odd dimensions, as nothing of the sort happens in even dimensions.

This may be attributed to the fact that for a CDL instanton solution, only the bubble size $\frac{1}{\cosh X_0}$ is specified [10]. That is, if $X_0 = a(> 0)$ is a good instanton solution, so is $X_0 = -a$. The only difference between the two solutions is that the former has a smaller portion of dS in it. If we assume some kind of duality between the field theories with the two instanton solutions as their backgrounds, $X_0 = 0$ would be a fixed point of the theory. Why this stands out only in odd dimensions is not clear at the moment.

7.2 Outlook

Although the graviton propagator written out in section 5.1 and the scalar propagator written out in section 6.8 doesn't provide any conclusive evidence of a holographic duality of two theories we can expect to fathom, assuming the latter certainly gives rise to many exciting possibilities.

If indeed such a correspondence were established, we will be able to gain a route to access a very novel kind of field theory; that is, one on Euclidean space with two CFTs (one possibly containing gravity) coupled in a rather peculiar way. This theory would have a tunable parameter, and might have a phase transition in odd dimensions.

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A The asymptotic behavior of $\mathcal{R}$ for odd dimensions

In order to examine the the poles of $\mathcal{R}$ in the limit $k \to -i\infty$ it is convenient to consider the asymptotic behavior of $\sin \pi x F(-N, N, 1 + x, t)$ in the limit $x \to -\infty$ where we have

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cancelled all the poles of the hypergeometric function by the multiplication of the sine function. This is because we are interested in the imaginary poles of $F(-N+1, N+1; 1-ik; t)/F(-N, N; 1-ik; t)$ for $k \to -i\infty$ and we know that the denominator gets rid of the poles, $ik =$ integer coming from the numerator, and hence our interest lie in the zeros of $\sin \pi x F(-N, N, 1+x, t)$.

We use the relations,

$$F(a, b; c; z) = (1-z)^{c-a-b} F(c-a, c-b; c; z) \tag{A.1}$$

$$\Gamma(z)\Gamma(1-z) = \pi \csc \pi z \tag{A.2}$$

and

$$\begin{aligned} F(a, b; c; z) = & \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F(a, b; a+b-c+1; 1-z) \\ & + (1-z)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F(c-a, c-b; c-a-b+1; 1-z) \end{aligned} \tag{A.3}$$

to obtain

$$\begin{aligned} & \sin \pi x F(-N, N; 1+x; t) = \\ & \quad \frac{\Gamma(-x-N)\Gamma(-x+N)}{\Gamma(-x)^2} \left[\left(\frac{t}{1-t}\right)^{-x} \left(\frac{1-t}{-x}\right) N \sin \pi N F(1+N, 1-N; 1-x; t) \right. \\ & \quad \left. + \sin \pi x F(-N, N; -x; 1-t) \right] \end{aligned} \tag{A.4}$$

First note that for $x \to -\infty$

$$\Gamma(-x+a)\Gamma(-x-a)/\Gamma(-x)^2 \approx 1 \tag{A.5}$$

for any fixed real number a. Also in this limit,

$$F(a, b; -x; z) = 1 + \mathcal{O}\left(\frac{1}{|x|}\right) \tag{A.6}$$

so up to leading order in $1/|x|$ we get,

$$\sin \pi x F(-N, N; 1+x; t) \approx \left(\frac{t}{1-t}\right)^{-x} \left(\frac{1-t}{-x}\right) N \sin \pi N + \sin \pi x \tag{A.7}$$

In the case $t/(1-t) \leq 1$ we see that the first terms in this equation vanishes in the desired limit. For $t/(1-t) > 1$, the second term becomes irrelevant.

Hence we can write the asymptotic behavior for our function in the limit $x \to -\infty$ as the following.

$$\sin \pi x F(-N, N; 1+x; t) \approx \begin{cases} [(1-t)N \sin \pi N] \frac{(t/(1-t))^{-x}}{-x} & t > 1/2 \\ \sin \pi x & t \leq 1/2 \end{cases} \tag{A.8}$$

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Note that we expect an infinite number of real zeros in x of $F(-N,N;1+x;t)$ for $t \le 1/2$ where for $t > 1/2$ the number of real zeros becomes finite.

Now note that since $\mathcal{R}$ is analytic for general t, for a given neighborhood of such t, the number of poles should be the same. Since $\mathcal{R}$ has an infinite number of imaginary poles as $k \to -i\infty$ for $t \le 1/2$, we know that the number of poles of $\mathcal{R}$ in the lower half plane of k should be infinite for a given neighborhood around $t = 1/2$. But we now also know from the asymptotic behavior of $\sin \pi x F(-N,N,1+x,t)$ that $\mathcal{R}$ has a finite number of poles on the lower imaginary axis. Hence $\mathcal{R}$ has an infinite number of poles that aren't imaginary in the lower half plane for $1/2 < t < 1/2 + \epsilon$ for some $\epsilon > 0$.

One might question the validity of this argument by questioning the statement that $\mathcal{R}(ix)$ has an infinite number of real poles at $t = 1/2$. Since $t = 1/2$ is a marginal value, one might feel that the argument based on the $x \to -\infty$ behavior of the function might not hold up. That is, it is possible that as $t \to 1/2-$, $x_M > 0$ for which at $x < -x_M$ we may safely approximate $\sin \pi x F(-N,N;1+x;t) \approx \sin \pi x$ might tend to infinity which would render the previous argument invalid.

Fortunately, we can explicitly prove that $\mathcal{R}(ix)$ has an infinite number of real poles for $t = 1/2$, which goes like the following. Let's deal with $\mathcal{R}$ directly for simplicity.

We write,

$$\mathcal{R}(ix) = \frac{N(1-t)F(-N+1,N+1;1+x;1/2)}{(x-N)F(-N,N;1+x;1/2)} \tag{A.9}$$

For sake of convenience, we will prove the equivalent statement that,

$$f(x) \equiv \frac{N}{4}\frac{F(-N+1,N+1;1+x;1/2)}{F(-N,N;1+x;1/2)} \tag{A.10}$$

has an infinite number of real poles.

Using the equalities,

$$\begin{aligned} F(-N+1,N+1;1+x;1/2) &= \frac{x+N}{N}F(-N,N+1;1+x;1/2) \\ &\quad -\frac{x-N}{N}F(-N+1,N;1+x;1/2) \end{aligned} \tag{A.11}$$

$$\begin{aligned} F(-N,N;1+x;1/2) &= \frac{1}{2}F(-N,N+1;1+x;1/2) \\ &\quad +\frac{1}{2}F(-N+1,N;1+x;1/2) \end{aligned} \tag{A.12}$$

and

$$F(a,1-a;1+x;1/2) = 2^{-x}\pi^{1/2}\frac{\Gamma(1+x)}{\Gamma\left(\frac{1}{2}a+\frac{1}{2}x+\frac{1}{2}\right)\Gamma\left(-\frac{1}{2}a+\frac{1}{2}x+1\right)} \tag{A.13}$$

we get,

$$f(x) \equiv \frac{\frac{1}{\Gamma(a+1/2)\Gamma(a+N)} - \frac{1}{\Gamma(a)\Gamma(a+N+1/2)}}{\frac{1}{\Gamma(a+1/2)\Gamma(a+N+1)} + \frac{1}{\Gamma(a+1)\Gamma(a+N+1/2)}} \tag{A.14}$$

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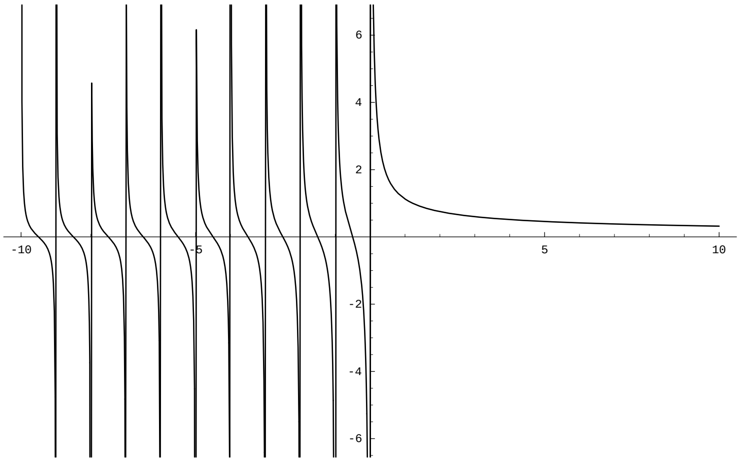

Figure 7. The plot a vs. $g(a)$.

where $a = \frac{1}{2}(x - N)$. Note that written in this way, both the numerator and denominator are analytic functions with no poles in the x plane. In order find the poles of $f(x)$, all we have to do is find the zeros of the denominator that aren't cancelled by a zero of the numerator.

Define the function,

$$g(a) \equiv \frac{\Gamma(a)}{\Gamma(a + 1/2)} \tag{A.15}$$

Then the zeros of the numerator come from the equation,

$$g(a) = g(a + N) \tag{A.16}$$

and the zeros of the denominator come from,

$$g(a + 1/2) = -g(a + N + 1/2) \tag{A.17}$$

From the analytic property of $\Gamma(a)$, we can infer that of $g(a)$. To sum up, $g(a)$ has the following properties.

1. For $a > 0$, $g(a)$ monotonically decreases from $+\infty$ at $a = 0+$ to 0 as $a \to \infty$.
2. For negative integer n, $g(a)$ monotonically decreases in the interval $(n, n + 1)$ from $g(n + 0) \to \infty$ to $g(n + 1 - 0) \to -\infty$.
3. For negative integer n, $g(n + 1/2) = 0$.

These facts are evident in figure 7.

Hence for half integer $N = M + 1/2$, there are M roots to $g(a) = g(a + N)$ each in the interval, $(n, n + 1/2)$ for $n = -1, \ldots, -M$.

Let's get to $g(b) = -g(b + N)$. First of all, there is one root in each interval $(n + 1/2, n + 1)$ for negative integer n. Also, there is one root in each interval $(n, n + 1/2)$ for integer $n < -M$.

Translating this for $a = b - 1/2$, the roots are given as the following.

1. There is one root in each interval $(n, n + 1/2)$ for negative integer n.

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2. There is one root in each interval $(n - 1/2, n)$ for integer $n < -M$.

Hence we see that there are an infinite number of negative zeros appearing in every 1/2 length interval in the denominator that aren't cancelled by zeros of the numerator. This completes the proof.

B The explicit expression for $w^I(\alpha_p)$ and Q_p

Defining α_p as,

$$\alpha_p(z) = F\left(\frac{D+2}{2} + ip, \frac{D+2}{2} - ip; \frac{D+3}{2}; 1-z\right) \tag{B.1}$$

we get,

$$\begin{aligned}
w^1(\alpha_p) =& \frac{4(D-2)}{D(D-3)}\left[\left(p^2 - \frac{3}{4}D^2 + 2D + 1\right) z(z-1) - \frac{D(D-3)}{4}\right]\alpha_p(z) \\
&+ \frac{8(D-2)}{D(D+3)}\left(p^2 + (\frac{D+2}{2})^2\right) z\left(z - \frac{1}{2}\right)(z-1)\beta_p(z) \\
w^2(\alpha_p) =& (1-z)\left[\frac{2(D-2)^2}{D(D-3)}\left(p^2 - \frac{3}{4}D^2 + 2D + 1\right) z + (D-1)(D-2)\right]\alpha_p(z) \\
&- \frac{4(D-2)^2}{D(D+3)}\left(p^2 + \left(\frac{D+2}{2}\right)^2\right) z(z-1)\left(z - \frac{D-1}{D-2}\right)\beta_p(z) \\
w^3(\alpha_p) =& \left[-\frac{2(D-2)^2}{D(D-3)}\left(p^2 - \frac{3}{4}D^2 + 2D + 1\right) z(z-1) + \frac{(D-1)(D-2)}{2}\right]\alpha_p(z) \\
&- \frac{4(D-2)^2}{D(D+3)}\left(p^2 + \left(\frac{D+2}{2}\right)^2\right) z\left(z - \frac{1}{2}\right)(z-1)\beta_p(z)
\end{aligned} \tag{B.2}$$

where β_p is defined as,

$$\beta_p(z) = -\frac{(D+3)/2}{(N+2)^2 + p^2}\frac{d\alpha_p(z)}{dz} \tag{B.3}$$

Also,

$$Q_p = \left[\frac{i\Gamma(D/2)D(D-3)}{4\pi^{D/2}(D-2)!(D-2)^2(D^2-1)}\right]\frac{p(p^2 + (N+1)^2)\Gamma(-ip + N - 1)}{\Gamma(-ip - N + 2)} \tag{B.4}$$

C Transverse traceless tensor propagators in $H^{(D-1)}$

We wish to examine the traceless spin 2 particle propagator on H^{D-1} with mass m. Note that we know from AdS/CFT that this taken to the boundary corresponds to the propagator of symmetric traceless tensor operators with dimension $\Delta = N + \sqrt{N^2 + m^2}$ (see, for example, [17]).

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The equation for the propagator for a massive traceless spin 2 particle in $H^{(D-1)}$ can be derived from the action,

$$\int d^{D-1}\sqrt{g}\left(\mathrm{R}-2\Lambda+\frac{1}{2}m^2h^{ij}h_{ij}\right) \tag{C.1}$$

where $g_{ij}=\gamma_{ij}+h_{ij}$ with the H^{D-1} metric γ_{ij}. R is the Ricci scalar for the metric g_{ij} We only focus on the traceless part of the spin 2 tensor for now, for reasons that will soon be clear. All indices are raised and lowered by the background metric.

We work with the background curvature radius, $R^2=-1$. Then the Ricci scalar of the background is given to be $-(D-1)(D-2)$, and the cosmological constant would be $-\frac{1}{2}(D-2)(D-3)$.

This can be perturbed to give the equations of motion ([20]),

$$\begin{aligned}\Box h_{ij}+g_{ij}\nabla^k\nabla^l h_{kl}-\nabla_j\nabla_k h_i^k-\nabla_i\nabla_k h_j^k&\\ +2R_{i\ j}^{\ k\ l}h_{kl}+2R_i^k h_{kj}-Rh_{ij}+2\Lambda h_{ij}&=m^2h_{ij}\end{aligned} \tag{C.2}$$

where the covariant derivatives, the Ricci tensors/scalar and the Riemann tensors all are given with respect to the background metric.

The l.h.s. of this equation has zero divergence. This can be seen by explicit calculation, or from the Bianchi identity. Hence for massive tensors, the transverseness of the propagator would not be a gauge condition, it would be a constraint coming from the equation of motion.

Using the transverseness of h_{ij}, the equation of motion reduces to

$$(\Box+2-m^2)h_{ij}=0 \tag{C.3}$$

The equation for the propagator can be obtained to be,

$$\left(-\widetilde{\Box}_1-2+m^2\right)G_{M\ \ i'j'}^{ij}(l(\mathcal{H}_1,\mathcal{H}_2),m^2)=\frac{1}{\sqrt{\gamma}}\left(\gamma_{(i'}^{(i}\gamma_{j')}^{j)}-\frac{1}{D-1}\gamma^{ij}\gamma_{i'j'}\right)\delta(\mathcal{H}_1,\mathcal{H}_2) \tag{C.4}$$

with the constraint,

$$\nabla_a G_{M\ \ i'j'}^{ij}(l(\mathcal{H}_1,\mathcal{H}_2),m^2)=0 \tag{C.5}$$

Note that the delta function on the righthand side of the equation for the propagator is not projected to be transverse, so it is actually zero for distinct $\mathcal{H}_1$ and $\mathcal{H}_2$.

We can solve this by following the steps sketched in [14], where first we solve,

$$\left(-\widetilde{\Box}_1-2+m^2\right)G_{M\ \ i'j'}^{ij}\left(l(\mathcal{H}_1,\mathcal{H}_2),m^2\right)=0 \tag{C.6}$$

for the maximally symmetric bitensor but now take the solution most singular at $l=0$ and obtain the multipicative constant by comparing it to the flat limit.

This can be done via the exact same procedure we obtained $W_{(p)i'j'}^{ij}$, but we impose different boundary conditions as we are working in a non-compact space. The solution is,

$$G_{M\ \ i'j'}^{ij}(l,m^2)=A(m^2)w^I\left(a_{i\sqrt{N^2+m^2}}\right)t_{I\ \ i'j'}^{ij}|_{z=\cosh^2\frac{l}{2}} \tag{C.7}$$

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where we define,

$$a_p(z) = \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip} F\left(\frac{D+2}{2} - ip, \frac{1}{2} - ip; 1-2ip; \frac{1}{z}\right) \tag{C.8}$$

$A(m^2)$ is some constant and $t^{ij}_{I\ i'j'}$ and $w^I(a_p)$ are given by (3.47), (3.48), (3.49), and (B.2).

From the fact that

$$a_p(z) \sim \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip} \qquad \text{for } z \to \infty \tag{C.9}$$

we see that for $z \to \infty$

$$w^I(a_p)t^{ij}_{I\ i'j'} \sim \left(\frac{1}{z}\right)^{N-ip} t^{ij}_{\ \ i'j'} \tag{C.10}$$

Hence we notice that the scaling dimension of $w^I(a_{ix})t^{ij}_{I\ i'j'}$ is $\Delta = N + x$. This can be seen by writing the geodesic length in H^{D-1} in Poincare coordinates. If we write the metric as,

$$ds^2 = \frac{dz^2 + dx_1^2 + \cdots + dx_{D-2}^2}{z^2} \tag{C.11}$$

the length of the geodesic connecting the two points $(z, \vec{x})$ and $(z', \vec{x'})$ is given as,

$$\cosh^2\frac{l}{2} = \frac{(z+z')^2 + (x-x')^2}{zz'} \tag{C.12}$$

so in the limit, $z, z' \to 0$,

$$w^I(a_{ix})t^{ij}_{I\ i'j'} \sim \left(\cosh^2\frac{l}{2}\right)^{-N-x} t^{ij}_{\ \ i'j'} \sim z^{N+x}z'^{N+x}(x-x')^{-2N-2x}t^{ij}_{\ \ i'j'} \tag{C.13}$$

We wish to extend the propagator $G^{ij}_{M\ i'j'}$ so it could have a general scaling dimension. But as can be seen from the expression (C.7), a massive H^{D-1} propagator with mass m has dimension, $\Delta = N + \sqrt{N^2+m^2}$. Hence the pieces with dimension $\Delta < N$ can't possibly be written in terms of massive propagators.

Also $A(m^2)$ exhibits singular behavior (hence forbidding the propagator of having certain scaling dimensions) if we try to generalize (C.7) by replacing $i\sqrt{N^2+m^2}$ by $i(\Delta - x)$. $A(\Delta)$ is evaluated up to a trivial multiplicative factor explicitly in appendix D, and we will address relevant issues there.

The important conclusion is that we will define the "generalized Green function"

$$G^{ij}_{H\ i'j'}(l,\Delta) = w^I(a_{i(\Delta-N)})t^{ij}_{I\ i'j'}|_{z=\cosh^2\frac{l}{2}} \tag{C.14}$$

that is, as the maximally symmetric bitensor with definite scaling dimension Δ. We note that,

$$G^{ij}_{H\ i'j'}(l,\Delta) \sim C(\Delta-2N)(\Delta-2N+1)e^{-\Delta l}t^{ij}_{\ \ i'j'} + \mathcal{O}\left(e^{-(\Delta+2)l}\right) \tag{C.15}$$

$$\sim C(\Delta-2N)(\Delta-2N+1)\frac{z^\Delta z'^\Delta}{|x-x'|^{2\Delta}}t^{ij}_{\ \ i'j'} + \mathcal{O}\left(\frac{z^{\Delta+2}z'^{\Delta+2}}{|x-x'|^{2\Delta+4}}\right) \tag{C.16}$$

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for all non-problematic(we will shortly explain what we mean by 'problematic') Δ. Also,

$$G^{ij}_{H\ i'j'}(l, \Delta) \propto G^{ij}_{M\ i'J'}(l, \Delta(\Delta - 2N)) \tag{C.17}$$

for $\Delta > N,\ \Delta \neq 2N$.

One thing we must note is that $G^{ij}_{H\ i'j'}(l, 2N)$ is not a propagator for a spin 2 tensor with $m^2 = 0$. This is because that the equation for the transverse traceless massless spin 2 propagator is,

$$\left(-\widetilde{\Box}_1 - 2\right) G^{ij}_{M\ i'j'}(l(\mathcal{H}_1, \mathcal{H}_2), 0) = \frac{1}{\sqrt{\gamma}} \delta^{ij}_{\ \ i'j'}(\mathcal{H}_1, \mathcal{H}_2) \tag{C.18}$$

where the delta function on the r.h.s. is a delta function projected on to transverse-traceless modes, so it is not zero for distinct $\mathcal{H}_1, \mathcal{H}_2$ in general. This situation arises because the transverseness of the propagator doesn't come from the equation of motion and has to be imposed as a gauge condition. This propagator is written out in a form compatible with our formalism in [21].

One more thing we have to be concerned about is that a_p(and hence $G^{ij}_{H\ i'j'}(l, N - ip)$) is singular for $1 - 2ip = -2n + 1$ for positive n. We are spared from some worry because in the case, $1 - 2ip = -2n$ we get,

$$a_p(z) = \left(\frac{1}{z}\right)^{\frac{(D+1)}{2} - n} F\left(\frac{D+1}{2} - n, -n; -2n; \frac{1}{z}\right) \tag{C.19}$$

so the hypergeometric function becomes a polynomial, stopping short of the divergent piece. So we just concern ourselves with the case, $p = -in$ for positive integer n.

In appendix E we will show that by expanding around $p = -in$, we can write,

$$\begin{aligned} G^{ij}_{H\ i'j'}(l, N - ip) &= \frac{1}{p + in} K_{-1,n} G^{ij}_{H\ i'j'}(l, N + n) && \text{(C.20)}\\ &\quad + H^{ij}_{0\ i'j'}(l, N - n) + (p + in) H^{ij}_{1\ i'j'}(l, N - n) && \text{(C.21)}\\ &\quad + \mathcal{O}((p + in)^2) && \text{(C.22)} \end{aligned}$$

and that for large l,

$$\begin{aligned} H^{ij}_{0\ i'j'}(l, N - n) &\sim e^{-(N-n)l} t^{ij}_{\ \ i'j'} && \text{(C.23)}\\ H^{ij}_{1\ i'j'}(l, N - n) &\sim l e^{-(N-n)l} t^{ij}_{\ \ i'j'} && \text{(C.24)} \end{aligned}$$

D The graviton propagator in flat space

The graviton propagator in flat space can be obtained by

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{i}{k^2 + m^2} \left(-\delta^{(a}_{(a'} \delta^{b)}_{b')} + \frac{2}{D-2} \eta^{ab} \eta_{a'b'} - \frac{2(D-3)}{D-2} \frac{k^a k^b k_{a'} k_{b'}}{m^4} \right.$$
$$\left. + \frac{2}{D-2} \frac{k^a k^b \eta_{a'b'} + \eta^{ab} k_{a'} k_{b'}}{m^2} - \frac{k^{(a} \delta^{b)}_{(a'} k_{b')}}{m^2} \right)$$

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This can be written in the form (C.14). For $f(x) \equiv \frac{m^{(D-3)/2}}{x^{(D-3)/2}} K_{(D-3)/2}(mx)$,

$$w^1_{\text{flat}}(l) \propto f(l) + \frac{2(D-3)}{m^4(D-2)}\left(\frac{f'(l)}{l^3} - \frac{f''(l)}{l^2}\right) - \frac{4}{m^2(D-2)}\frac{f'(l)}{l} \tag{D.1}$$

up to a constant independent of mass. For $l \to 0$,

$$w^1_{\text{flat}}(l) \sim \frac{1}{m^4}\frac{1}{l^{D+1}} \tag{D.2}$$

up to a constant independent of mass.

Now for $l \to 0$ since

$$a_p(z) \sim \frac{\Gamma(1-2ip)\Gamma((D-1)/2)}{\Gamma\left(\frac{D+2}{2} - ip\right)\Gamma\left(\frac{1}{2} - ip\right)}\left(\frac{1}{l}\right)^{(D-1)} \tag{D.3}$$

we obtain

$$w^1_p(l) \sim \frac{\Gamma(1-2ip)}{\Gamma\left(\frac{D+2}{2} - ip\right)\Gamma\left(\frac{1}{2} - ip\right)}\left(\frac{1}{l}\right)^{(D+1)} \tag{D.4}$$

up to a constant independent of p.

Comparing these two values for a given mass(with $p = i\sqrt{N^2+m^2}$), we obtain up to a non-singular constant,

$$\begin{aligned} A(m^2) &\propto \frac{1}{m^4}\frac{\Gamma\left(\frac{D+2}{2} + \sqrt{N^2+m^2}\right)\Gamma\left(\frac{1}{2} + \sqrt{N^2+m^2}\right)}{\Gamma(1+2\sqrt{N^2+m^2})} \\ &\propto \frac{1}{m^4}\frac{\Gamma\left(\frac{D+2}{2} + \sqrt{N^2+m^2}\right)}{\Gamma(1+\sqrt{N^2+m^2})} \end{aligned} \tag{D.5}$$

Trying to generalize this for a general scaling dimension we get,

$$A(\Delta) \propto \frac{1}{(\Delta(\Delta-2N))^2}\frac{\Gamma(\Delta+2)}{\Gamma(\Delta-N+1)} \tag{D.6}$$

This is singular for $\Delta = 0,\ 2N$ and $-n-1$ for positive integer n. Also note that this is zero for $\Delta = N-n$ for positive integer n. This leads to the interesting fact that due to (C.20),

$$\lim_{\Delta \to N-n} A(\Delta) G^{ab}_{H\ a'b'}(l, \Delta) \propto G^{ab}_{H\ a'b'}(l, N+n) \tag{D.7}$$

for positive integer n.

E Deconstructing singular tensor propagators

We deal with the singularity of $G^{ij}_{H\ i'j'}(l,\Delta)$ at $\Delta = N-n$ for positive integer n by writing,

$$\begin{aligned} a_p(z) = &\left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip} f(p, 2n-1, z) \\ &+ \frac{1}{p+in}\left(\frac{i}{2}\right)\frac{\left(\frac{(D+2)}{2} - ip\right)_{2n}\left(\frac{1}{2} - ip\right)_{2n}}{2n!(1-2ip)_{2n-1}} \\ &\times \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip+2n} F\left(\frac{D+2}{2} - ip + 2n, \frac{1}{2} - ip + 2n; 2n+1; \frac{1}{z}\right) \end{aligned} \tag{E.1}$$

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where we have defined,

$$(x)_n \equiv x(x+1)\cdots(x+n-1) \tag{E.2}$$

and $f(p,m,z)$ is the polynomial

$$f(p,m,z) \equiv \sum_{n=0}^{m} \frac{1}{n!} \frac{((D+2)/2-ip)_n(1/2-ip)_n}{(1-2ip)_n} \left(\frac{1}{z}\right)^n \tag{E.3}$$

To put this in a form which is more useful, we expand the latter part of a_p around $p=-in$ for which we get,

$$\begin{aligned}
a_p(z) =& \frac{1}{p+in} K_{-1,n} a_{in}(z) \\
&+ \left[\left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-n} f_{-in}(z) + K_{0,n} a_{in}(z) + L_{0,n} c_{in}(z) \right] \\
&+ (p+in) \left[(I_{1,n} \ln z + J_{1,n}) \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-n} f_{-in}(z) + K_{1,n} a_{in}(z) \right. \\
&\qquad \left. + L_{1,n} c_{in}(z) + M_{1,n} d_{in}(z) \right] \\
&+ \mathcal{O}((p+in)^2) \\
\equiv& \frac{1}{p+in} K_{-1,n} a_{in}(z) + h_{0,-in}(z) + (p+in) h_{1,-in}(z) \\
&+ \mathcal{O}((p+in)^2)
\end{aligned} \tag{E.4}$$

Where we have conveniently defined,

$$f_{-in}(z) \equiv f\left(-in, 2n-1, \frac{1}{z}\right) \tag{E.5}$$

$$c_{in}(z) \equiv \frac{\partial}{\partial p} \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip+2n} F\left(\frac{D+2}{2}-ip+2n, \frac{1}{2}-ip+2n; 2n+1; \frac{1}{z}\right)\Bigg|_{p=-in} \tag{E.6}$$

$$d_{in}(z) \equiv \frac{\partial^2}{\partial p^2} \left(\frac{1}{z}\right)^{\frac{(D+2)}{2}-ip+2n} F\left(\frac{D+2}{2}-ip+2n, \frac{1}{2}-ip+2n; 2n+1; \frac{1}{z}\right)\Bigg|_{p=-in} \tag{E.7}$$

We can finally write,

$$\begin{aligned}
G^{ij}_{H\ i'j'}(l, N-ip) =& \frac{1}{p+in} K_{-1,n} G^{ij}_{H\ i'j'}(l, N+n) \\
&+ H^{ij}_{0\ i'j'}(l, N-n) + (p+in) H^{ij}_{1\ i'j'}(l, N-n) \\
&+ \mathcal{O}((p+in)^2)
\end{aligned} \tag{E.8}$$

where

$$H^{ij}_{0\ i'j'}(l, N-n) \equiv w^I(h_{0,-in}(z)) t^{ij}_{I\ i'j'} \tag{E.9}$$

$$H^{ij}_{1\ i'j'}(l, N-n) \equiv w^I(h_{1,-in}(z)) t^{ij}_{I\ i'j'} \tag{E.10}$$

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Note that for $l \to \infty$ (since $n > 0$),

$$h_{0,-in}(z) \sim e^{-\left(\frac{(D+2)}{2}-n\right)l} \tag{E.11}$$

$$h_{1,-in}(z) \sim le^{-\left(\frac{(D+2)}{2}-n\right)l} \tag{E.12}$$

and hence,

$$H_{0\ i'j'}^{ij}(l, N-n) \sim e^{-(N-n)l} t^{ij}_{\ \ i'j'} \tag{E.13}$$

$$H_{1\ i'j'}^{ij}(l, N-n) \sim le^{-(N-n)l} t^{ij}_{\ \ i'j'} \tag{E.14}$$

F Degenerate modes of the graviton

In this section, we will identify the degenerate modes of the transverse-traceless graviton propagator in H^{D-1}.

We start with the scalar mode, $E^{(p)}$ such that,

$$\tilde{\Box} E^{(pv)} = -(N^2 + p^2) E^{(pv)} \tag{F.1}$$

In H^{D-1} we find that,

$$\tilde{\Box}\left(\tilde{\nabla}_i \tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box}\right) E^{(pv)} = (-N^2 - p^2 - 2(2N+1))\left(\tilde{\nabla}_i \tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box}\right) E^{(pv)} \tag{F.2}$$

Also,

$$\left(\tilde{\nabla}^i \tilde{\nabla}_i - \frac{\delta^i_i}{D-1}\tilde{\Box}\right) E^{(pv)} = 0 \tag{F.3}$$

$$\tilde{\nabla}^i\left(\tilde{\nabla}_i \tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box}\right) E^{(pv)} = \frac{D-2}{D-1}(-N^2 - p^2 - (2N+1))\tilde{\nabla}_j E^{(p)} \tag{F.4}$$

Hence $(\tilde{\nabla}_i \tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box})E^{(pv)}$ is a symmetric transverse traceless spin 2 mode for $p = i(N+1)$, and hence its eigenvalue with respect to $\tilde{\Box}$ would be $-2N-1$. Therefore this is degenerate with the spin 2 modes $r_{ij}^{(pu)}$ (whose eigenvalues are given by $-(N^2+2+p^2)$) with $p = i(N-1)$.

For the vector mode, $F_i^{(pw)}$ such that,

$$\tilde{\Box} F_i^{(pw)} = -(N^2 + p^2 + 1) F_i^{(pw)} \tag{F.5}$$

we find in H^{D-1},

$$\tilde{\Box} F_{(i|j)}^{(pw)} = (-N^2 - p^2 - 1 - (2N+2)) F_{(i|j)}^{(pw)} \tag{F.6}$$

Also since $F_i^{(pv)}$ are transverse,

$$F_{(i|i)}^{(pv)} = 0 \tag{F.7}$$

$$\tilde{\nabla}^i F_{(i|j)}^{(pw)} = (-N^2 - p^2 - 1 - 2N) F_j^{(pw)} \tag{F.8}$$

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So $F^{(pw)}_{(i|j)}$ is a symmetric transverse traceless spin 2 mode for $p = i(N+1)$, and hence its eigenvalue with respect to $\tilde{\Box}$ would be -2. Therefore this is degenerate with the spin 2 modes $r^{(pu)}_{ij}$ (whose eigenvalues are given by $-(N^2+2+p^2)$) with $p = iN$.

Now let's show that all $r'^{(i(N-1)u)}_{ij}$ come from $E^{(i(N+1)v)}$ and that all $r'^{((iN)u)}_{ij}$ come from $F^{(i(N+1)w)}_i$ where $r'^{((p)u)}_{ij}$ are defined by (4.15).

Note that by the form of $(\tilde{\nabla}_i\tilde{\nabla}_j - \frac{\tilde{\gamma}_{ij}}{D-1}\tilde{\Box})E^{(pv)}$, this has even parity, and hence this certainly cannot saturate $\{r^{(pu)}_{ij}\}$. But our objective would be to get rid of the modes contributing to $W^{ij}_{(p)i'j'}$ with $p = iN, i(N-1)$ in our propagator and as will be shown, this can be done.

Define,

$$Z_{(p)} = \sum_v E^{(pv)\dagger}(\mathcal{H})E^{(pv)}(\mathcal{H}') \tag{F.9}$$

$$Z^i_{(p)i'} = \sum_v F^{(pv)i\dagger}(\mathcal{H})F^{(pv)}_{i'}(\mathcal{H}') \tag{F.10}$$

for properly normalized, regular $E^{(pv)}$ and $F^{(pw)}_i$. These are maximally symmetric bitensors as they are invariant under any isometries. Also, they show regular behavior at $\mathcal{H} = \mathcal{H}'$, i.e. the coincident point. A covariant derivative of a maximally symmetric bitensor is also a maximally symmteric bitensor, hence so are,

$$Z^{ij}_{1(p)i'j'} = \left(\tilde{\nabla}^i\tilde{\nabla}^j - \frac{\tilde{\gamma}^{ij}}{D-1}\tilde{\Box}\right)\left(\tilde{\nabla}_{i'}\tilde{\nabla}_{j'} - \frac{\tilde{\gamma}_{i'j'}}{D-1}\tilde{\Box}\right)Z_{(p)} \tag{F.11}$$

$$Z^{ij}_{2(p)i'j'} = Z^{(i|j)}_{(p)(i'|j')} \tag{F.12}$$

From the mode sum and by the behavior of the individual modes for $p = i(N+1)$, $Z^{ij}_{1(i(N+1))i'j'}$ and $Z^{ij}_{2(i(N+1))i'j'}$ are symmetric, transverse, traceless maximally symmetric bitensors behaving regularly at the coincident point, which satisfy,

$$\tilde{\Box}Z^{ij}_{1(i(N+1))i'j'} = -(N^2-(N-1)^2+2)Z^{ij}_{1(i(N+1))i'j'} \tag{F.13}$$

$$\tilde{\Box}Z^{ij}_{2(i(N+1))i'j'} = -(N^2-N^2+2)Z^{ij}_{2(i(N+1))i'j'} \tag{F.14}$$

so we see that,

$$Z^{ij}_{1(i(N+1))i'j'}(l) \propto W^{ij}_{(i(N-1))i'j'}(il) \tag{F.15}$$

$$Z^{ij}_{2(i(N+1))i'j'}(l) \propto W^{ij}_{(iN)i'j'}(il) \tag{F.16}$$

where $W^{ij}_{(p)i'j'}$ is defined in (3.45) and can be written alternatively as in (4.15). This is because the conditions mentioned are all that we used in obtaining $W^{ij}_{(p)i'j'}$ in the first place. (We have used $W^{ij}_{(p)i'j'}$ instead of $Z^{ij}_{(p)i'j'}$ here due to the fact that $Z^{ij}_{(p)i'j'}$ may have poles for the values concerned.) If indeed this is true for some non-zero proportionality constant, this means that the derivatives of $E^{(i(N+1)v)}$ and $F^{(i(N+1)w)}_i$ give all the modes $\{r'^{(i(N-1)u)}_{ij}\}$ and $\{r'^{((iN)u)}_{ij}\}$ respectively.

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The only potential problem lies in the fact that $Z^{ij}_{1(i(N+1))i'j'}$ and $Z^{ij}_{2(i(N+1))i'j'}$ might be zero. From [14] and we see that,

$$Z_{(p)} = C_p F\left(N+ip, N-ip; N+\frac{1}{2}; 1-z\right) \tag{F.17}$$

$$\begin{aligned} Z^i_{(p)i'} &= C'_p\left[\tilde{\gamma}^i_{i'}\left(\frac{2z(z-1)}{N}\frac{d}{dz}+(2z-1)\right)\right. \\ &\quad \left.+n^i n_{i'}\left(\frac{2z(z-1)}{N}\frac{d}{dz}+(2z-2)\right)\right]\gamma_p(z) \end{aligned} \tag{F.18}$$

$$\text{for} \qquad \gamma_p(z) \equiv F\left(N+1+ip, N+1-ip; N+\frac{3}{2}; 1-z\right) \tag{F.19}$$

and from [15] we see that

$$C_p \propto \frac{\left[p^2+(N-1)^2\right]\Gamma(ip+N-1)\Gamma(-ip+N-1)}{\Gamma(ip)\Gamma(-ip)} \tag{F.20}$$

$$C'_p \propto \frac{\left[p^2+N^2\right]\Gamma(ip+N-1)\Gamma(-ip+N-1)}{\Gamma(ip)\Gamma(-ip)} \tag{F.21}$$

up to a factor independent of p. Although C_p and C'_p have poles, by direct calculation, we can obtain non-zero, non-sigular $Z^{ij}_{1(i(N+1))i'j'}$ and $Z^{ij}_{2(i(N+1))i'j'}$.

Of course $Z^{ij}_{1(i(N+1))i'j'}$ and $Z^{ij}_{2(i(N+1))i'j'}$ can be obtained explicitly to verify (F.15) and (F.16).

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES];
O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [SPIRES].

[2] A. Strominger, *The dS/CFT correspondence*, *JHEP* **10** (2001) 034 [hep-th/0106113] [SPIRES].

[3] L. Dyson, J. Lindesay and L. Susskind, *Is there really a de Sitter/CFT duality*, *JHEP* **08** (2002) 045 [hep-th/0202163] [SPIRES].

[4] J.M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, *JHEP* **05** (2003) 013 [astro-ph/0210603] [SPIRES].

[5] B. Freivogel, Y. Sekino, L. Susskind and C.-P. Yeh, *A holographic framework for eternal inflation*, *Phys. Rev.* **D 74** (2006) 086003 [hep-th/0606204] [SPIRES].

[6] L. Susskind, *The census taker's hat*, arXiv:0710.1129 [SPIRES].

[7] R. Bousso et al., *Future foam*, *Phys. Rev.* **D 78** (2008) 063538 [arXiv:0807.1947] [SPIRES].

[8] J. Garriga and A. Vilenkin, *Holographic multiverse*, *JCAP* **01** (2009) 021 [arXiv:0809.4257] [SPIRES].

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[9] S.R. Coleman, *The fate of the false vacuum. 1. Semiclassical theory*, *Phys. Rev.* **D 15** (1977) 2929 [*Erratum ibid.* **D 16** (1977) 1248] [SPIRES]; C.G. Callan Jr. and S.R. Coleman, *The fate of the false vacuum. 2. First quantum corrections*, *Phys. Rev.* **D 16** (1977) 1762 [SPIRES].

[10] S.R. Coleman and F. De Luccia, *Gravitational effects on and of vacuum decay*, *Phys. Rev.* **D 21** (1980) 3305 [SPIRES].

[11] T. Hertog and N. Turok, *Gravity waves from instantons*, *Phys. Rev.* **D 62** (2000) 083514 [astro-ph/9903075] [SPIRES].

[12] S. Gratton and N. Turok, *Cosmological perturbations from the no boundary Euclidean path integral*, *Phys. Rev.* **D 60** (1999) 123507 [astro-ph/9902265] [SPIRES].

[13] S.W. Hawking, T. Hertog and N. Turok, *Gravitational waves in open de Sitter space*, *Phys. Rev.* **D 62** (2000) 063502 [hep-th/0003016] [SPIRES].

[14] B. Allen and T. Jacobson, *Vector two point functions in maximally symmetric spaces*, *Commun. Math. Phys.* **103** (1986) 669 [SPIRES].

[15] R. Camporesi and A. Higuchi, *Spectral functions and zeta functions in hyperbolic spaces*, *J. Math. Phys.* **35** (1994) 4217 [SPIRES].

[16] B. Allen, *Maximally symmetric spin two bitensors on S^3 and H^3*, *Phys. Rev.* **D 51** (1995) 5491 [gr-qc/9411023] [SPIRES].

[17] A. Polishchuk, *Massive symmetric tensor field on AdS*, *JHEP* **07** (1999) 007 [hep-th/9905048] [SPIRES].

[18] T. Tanaka and M. Sasaki, *The spectrum of gravitational wave perturbations in the one-bubble open inflationary universe*, *Prog. Theor. Phys.* **97** (1997) 243 [astro-ph/9701053] [SPIRES].

[19] J. Erdmenger and H. Osborn, *Conserved currents and the energy-momentum tensor in conformally invariant theories for general dimensions*, *Nucl. Phys.* **B 483** (1997) 431 [hep-th/9605009] [SPIRES].

[20] N.H. Barth and S.M. Christensen, *Quantizing fourth order gravity theories. 1. The functional integral*, *Phys. Rev.* **D 28** (1983) 1876 [SPIRES].

[21] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Graviton and gauge boson propagators in AdS_{d+1}*, *Nucl. Phys.* **B 562** (1999) 330 [hep-th/9902042] [SPIRES].

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